Generating Shock Scenarios for Risk-Based Capital Using a GARCH-VEC Model

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The views expressed are those of the individual authors and do not reflect official positions of the Federal Housing Finance Agency.

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Abstract

We present a process for scenario-based risk measurement, where portfolios may comprise significant non-linear risk exposures, such as mortgages or interest-rate options. The goal is a computationally efficient protocol for defining stress scenarios that is nonetheless comprehensive in its coverage of risk dimensions, and sufficiently stable statistically for a production context. The process employs time-series estimation – specifically, a generalized autoregressive heteroskedasticity (GARCH) vector error correction (VEC) model – to identify a concise set of shock scenarios with good coverage of the underlying state space, and it uses standard risk measurement technologies to generate stable risk exposure assessments. We find that the GARCH-VEC approach is able to generate plausible scenarios that are systematically distributed in a six-dimensional factor space, and that create significant stress for a hypothetical bank.

DCMI summary

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I. Introduction

This paper is motivated by a need for a new regulatory risk-based capital framework for the Federal Home Loan Banks (FHLBs). The process we consider here incorporates a diverse range of risk factors, tailored to hedged mortgage lenders, while potentially delegating the detailed portfolio analytics to the FHLBs, in the spirit of the proposed Basel II internal risk-based (IRB) approach. We present a process for scenario-based risk measurement and modeling in the context of multiple entities with diverse portfolios, where each portfolio may comprise significant non-linear risk exposures, such as mortgages and interest-rate options. This paper discusses our overall methodology primarily in the context of a multivariate case, but also compares our results to the benchmark case of no dependence among the state variables.

A. Context

Our goal is to find a computationally efficient protocol for defining stress scenarios with two characteristics: (a) it is comprehensive in its coverage of risk dimensions for the relevant portfolios, and (b) it is sufficiently stable statistically to be applied in a production context. Several aspects of this context are important for the structure of the proposed process.

First, the process must apply well to each of several diverse portfolios. That is, the scenario selection process should not depend on the particular composition of each portfolio, beyond broad restrictions on the types of exposures allowed. The process addresses this via a factor model, in which a concise set of exogenous fundamental risk factors define the stochastic inputs. Shock scenarios are calculated strictly in terms of these exogenous factors.

Second, the portfolios are allowed to be internally diverse. This research grows out of an ongoing regulatory program at the Federal Housing Finance Agency (FHFA) defining risk-based capital (RBC) requirements for the FHLBs.[1] In the empirical section below, we calibrate the portfolios to approximate the holdings of a stylized FHLB. FHLBs hold portfolios composed of

[1] See Federal Housing Finance Board (2001). FHFA's existing RBC methodology employs historical value at risk (VaR) from overlapping measurement windows dating back to 1978. This methodology has been criticized on a number of grounds, including the relatively large number of scenarios involved, and the restriction of state variables to interest-rate shocks. Relative to FHFA's legacy methodology, the approach presented in this paper would: (a) incorporate mortgage OAS risk; (b) incorporate mortgage prepayment model risk; (c) provide a more consistent measure of risk across FHLBs; (d) be more consistent with Basel Committee guidance; and (e) be less computationally burdensome for the FHLBs.

mortgages (including whole loans, mortgage-backed securities (MBS), and collateralized mortgage obligations (CMO)), advances (short-term money-market loans, frequently with embedded optionality), debt (including especially both simple and callable debentures), and swaps (also frequently callable). Other exposures include swaptions, deposit funding, U.S. Treasury and Agency debt, and interest-rate caps, floors, futures, forwards, and options. The exogenous risk factors are chosen to explain the majority of the observable variation for the security types with significant representation in the modeled institution.

Third, non-linear exposures are regarded as typical, effectively ruling out the use of a simple linear factor model to approximate portfolio risk. Instead, the process requires no explicit assumptions regarding the structure of the portfolios' sensitivities to the exogenous risk factors. We delegate the state-dependent valuation of positions to risk modeling systems specific to each portfolio. In the empirical example, we consider the mark-to-model valuations of a stylized FHLB using one such vendor model. In a production implementation, we anticipate that shock scenarios would be established independently of any particular portfolio, and that portfolio valuation and analysis under the shock scenarios would be delegated to the institution responsible for the portfolio, similar to the IRB approach of the Basel II proposal.

B. Basic Structure

We follow McNeil, Frey, and Embrechts (2005, §2.1) and Flood and Korenko (2008) in defining a portfolio value, V_t, as a function of time and a vector of exogenous risk factors, $\boldsymbol{u}_t \in \mathbb{R}^d$: $V_t \equiv f(t, \boldsymbol{u}_t)$, and losses, $\mathcal{L}_t$, as the inverse of changes in value:

$$\mathcal{L}_{t+1}(\boldsymbol{u}_t, \boldsymbol{w}_{t+1}) \equiv -[V_{t+1} - V_t] = -[f(t+1, \boldsymbol{u}_{t+1}) - f(t, \boldsymbol{u}_t)] = -[f(t+1, \boldsymbol{u}_t+\boldsymbol{w}_{t+1}) - f(t, \boldsymbol{u}_t)] \ , \qquad (1)$$

where the innovation in the risk factors, $\boldsymbol{w}_{t+1} \in \mathbb{R}^d$ is a shock scenario consisting of d simultaneous exogenous random shocks, conditional on the information, $\mathcal{F}_t$, available at time t. Randomness enters the model only through the exogenous risk factors, $\boldsymbol{w}_{t+1}$, due to the conditioning on $\mathcal{F}_t$.

Our focus is on modeling the exogenous shocks in $\boldsymbol{w}_t$, and understanding the net effect of those shocks on the ultimate valuation metrics, V_t. Our particular interest is in the risk profiles of the FHLBs, and we have chosen a set of risk factors tailored to their exposures. Some of these factors (e.g., the agency debt spread) are fairly specific to the FHLBs. However, several of these risk factors (e.g., interest rates) are of more general applicability, as is our overall methodology.

Note that we treat the valuation function $f(.)$ effectively as a black box. That is, we do not develop here a particular parameterized pricing model. Instead, because we confront a diverse array of possible security types, we delegate these technical details to a third-party software implementation that uses a variety of instrument-specific models.[2] We similarly take as given the managerial strategy vector which we reduce (simplistically) to a set of dollar portfolio weights invested in financial assets, liabilities, and derivatives to approximate a generic FHLB. We exclude the impact of strategy and valuation, $f(.)$ from our scope to focus on the issue of generating shock scenarios. Figure 1 depicts our general process for generating scenarios and analyzing their effects on a representative FHLB portfolio.

I. Literature Review

A. The Issues

The universe of possible exogenous factors, $\boldsymbol{w}_t$, is very broad, and many direct measures are closely correlated with each other. Furthermore, the relative importance of risk exposures at the level of bank capital may differ greatly from their absolute priorities if the institution has hedged out its most significant exposures, thus raising the relative prominence of the others. This is particularly true of the FHLBs, which frequently micro-hedge new exposures. In the empirical section below, we consider a set of exogenous factors appropriate for portfolios containing significant holdings of mortgages and interest-rate optionality.

B. Interest-Rate Modeling

For FHLB portfolios, we expect the relevant exogenous risk factors to directly or indirectly involve interest rates. Thus, our search for risk factors and our use of comparative approaches draws on the finance literature on term structure modeling. This literature is extensive, and a comprehensive review is well beyond the scope of this paper. Fortunately, there are several excellent recent surveys already available, including Dai and Singleton (2003), Piazzesi (2003), Ahn, Dittmar & Gallant (2002), and Gibson, Lhabitant & Talay (2001). Brandt

[2] Specifically, we use AppPort™, a package of risk analytics provided by PolyPaths Fixed Income Analytics to perform the transformation represented by $f(.)$.

and Chapman (2005) and Hughston (2003) each provide a shorter overview. Brigo & Mercurio (2001) focus on financial and trading applications of the models. Diebold, Piazzesi & Rudebusch (2005) examine the finance literature from a more traditional macroeconomic perspective.

Statistical modeling of interest rates is complicated by the obvious structural connections, enforced by arbitrage, among rates of different maturities. Within the term structure on a given date – i.e., within a given cross-section of interest rates – the long-term spot (i.e., implied zero-coupon) yields should equal average risk-adjusted expectations of future short rates. As rates change over time, this structural constraint forces rates of different tenors to move together (see Piazzesi (2003)). Defining and estimating the structure of this co-movement composes the bulk of the effort in the vast literature on term-structure modeling. There is a rough divergence of emphasis between cross-sectional no-arbitrage models on the one hand, and time-series models on the other. No-arbitrage models typically calibrate the parameters of a stochastic differential equation to the yield curve (and, in some cases, to the volatility surface) at a particular point in time. Derivative prices calculated relative to the calibrated term-structure dynamics are therefore arbitrage-free relative to one another, making this methodology a natural choice for trading applications. However, because time consistency is not enforced (e.g., by fitting the model to time-series data), parameter calibrations can shift from day to day, typically in violation of model assumptions. Ironically, calibrated no-arbitrage models can be internally inconsistent and may thus allow arbitrage opportunities in certain contingent claims (see, for example, Brandt & Yaron (2003), and Backus, Foresi & Zin (1998)).

There is a broad consensus, dating from the seminal paper by Litterman & Scheinkman (1991), that three random factors are adequate to explain the vast majority of the variability in yields over time. These factors are typically identified through a factor decomposition or principal components analysis of the covariance matrix of yield changes. Litterman & Scheinkman (1991) attribute these factors directly to the level, steepness, and curvature of the terms structure. In a companion article, Litterman, Scheinkman & Weiss (1991) link the curvature of the yield curve to the implied volatility of interest rates from futures options, and illustrate this link with a "butterfly portfolio" example (see also Christiansen & Lund (2005)). Collin-Dufresne & Goldstein (2002) reveal that bonds alone are unable empirically to span stochastic volatility risk. They proceed to show that most affine term structure models in current use, including all one- and two-factor models, are similarly incapable of generating – i.e., explaining – this sort of unspanned stochastic volatility (USV). They derive the necessary and sufficient conditions for a three-factor affine model to be able to generate and explain USV. In the current study, we consider three term-structure factors – long rate, short rate, and volatility – which we believe capture the vast majority of the variation in default-risk-free rates.

While the structure imposed by affine models has important benefits, especially in the area of arbitrage-free pricing of bonds and interest-rate derivatives, a number of recent papers suggest that this structure is more a hindrance than a help for forecasting purposes.[3] Duffee (2002), for example, concludes that forecasts from affine models underperform a random-walk forecast. This is significant, since we are particularly interested in forecasting the future density of our risk factors, rather than satisfying current-period no-arbitrage pricing conditions. Diebold and Li (2006) do not consider no-arbitrage models, but instead compare the forecasting performance of a number of different specifications of a Nelson-Siegel framework, and find that a relatively simple three-factor vector autoregression (VAR) provides superior forecasts at long horizons (i.e., one year).

C. Choice of Risk Factors

Based on our review of the literature on interest rate modeling, we conclude that the set of relevant risk factors should include measures of the position, slope and curvature of the yield curve. Specifically, we measure these features of the yield curve using a short-term interest rate, a long-term interest rate, and a measure of interest-rate implied volatility. In addition, we believe it is important to consider the relative cost of funding and the expected return from investing in mortgage-related investments. To measure the cost of funding, we use the LIBOR-Agency yield spread. Finally, we use two factors to measure the returns (exposures) from holding mortgage-related investments: the mortgage option-adjusted spread and the mortgage basis spread.[4] Thus, for the present analysis, we define the following six factors:

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[3] See, for example, Duffee (2008), Diebold and Li (2006), Duffee (2002), and Andersen, Bollerslev, Diebold, and Labys (2003).

[4] In production, we also intend to include sensitivity to changes in a prepayment model. However, we do not address the prepayment model here since it is not a factor in scenario generation, but rather a factor affecting f(.). Rather, it will be used to ensure that the Banks will hold levels of capital consistent with the standard prepayment model that provides the more adverse outcome.

- short-term interest rate (*RST*),
- long-term interest rate (*RLT*),
- interest-rate implied volatility (*VOL*),
- the LIBOR-Agency yield spread (*LAS*),
- mortgage option-adjusted spread calculated from current TBA prices (*OAS*), and
- mortgage basis spread (*BAS*).

This selection is based on an analysis of the research literature, as well as discussions with FHFA staff and other industry practitioners.

We used multivariate regression analysis to confirm that these factors are useful in explaining changes in the values of the FHLBs balance sheets.[5] First, changes in these six risk factors explain at least 92 percent of the price variation in Agency securities, Treasuries, mortgage-backed securities, and swaps, which are typical instruments that the FHLBs hold. Second, depending on the FHLB, these factors explain between 61 percent and 98 percent of the historical variation in the FHLB's market value of equity.

Our goal is to define a robust methodology for forecasting plausibly severe shock scenarios in each of the remaining six dimensions.

II. Data

Our daily data cover the period April 4, 1994 through May 5, 2008. We list the sources for the raw data in Table 1 and provide a raw data plot for each risk factor in Figure 2. In each case, visual inspection reveals substantial serial correlation and volatility clustering. This suggests that the data exhibit autocorrelation and ARCH effects. In addition, the persistent upward and downward trends of the series suggest that these data may be nonstationary. Indeed, as we discuss below, we find that all of our risk factor data have unit roots. Table 2a contains selected descriptive statistics for the risk factor data in levels. Given that our data are nonstationary, we provide more complete descriptive statistics of the stationary (differenced) data in Table 2b. We observe that, in particular, *LAS* exhibits substantial negative skew.[6] We also note that each of the risk factors is leptokurtic, and some are very substantially leptokurtic.[7]

[5] Our regressions include both first and second order terms, but no cross products.

[6] A distribution centered on zero with negative skew would tend to show frequent small positive and negative values and relatively fewer large negative values. That is, the left tail of the distribution is longer than the right tail.

[7] A leptokurtic distribution is more peaked than the normal distribution and will have "fat tails" relative to the normal distribution.

Thus, it does not appear that these data are drawn from normal distributions, and the Jarque-Bera test confirms this finding.

The structure of the model bank we employ appears in Table 3, along with some of the key modeling assumptions applied in the valuations. Assets consist of: (a) collateralized money-market loans, known as advances, to member banks and thrifts; (b) other money-market instruments and loans to housing finance authorities (HFAs); (c) fixed-rate whole-loan mortgages; and (d) mortgage securities (MBS and CMO). Liabilities consist of various forms of debt. Most FHLB debt is in the form of "consolidated obligations" (COs), which are marketed through a joint-issuance facility, the Office of Finance (OF). Finally, there are two swap portfolios hedging a portion of the advances and COs, respectively. The positions are chosen to mimic a "typical" asset-liability strategy of a FHLB. In particular, advances are micro-hedged with matching debt or swaps, while mortgages and mortgage securities are hedged on portfolio basis with tailored portfolios of callable debt. Otherwise unhedged debt is then swapped to LIBOR, or a similar short-term variable rate.

Overall, we have divided the model bank into 31 separate portfolios for modeling purposes. There are 17 asset portfolios, 12 liability portfolios, and two swap portfolios. Each portfolio subsumes a number of individual positions to be modeled. There are 233 total positions: 109 asset positions, 106 debt positions, and 18 swap positions. All positions within a given portfolio receive the same valuation treatment and pricing assumptions. Pricing assumptions can vary between portfolios, however. All valuations are made relative to market data for month-end of July, 2005.[8]

III. Shock Estimation

Given the risk factor data we assembled, our goal is to construct a factor model that will allow us to infer statistically their dependence structure. Our factor model should characterize the data generating processes (DGPs) for these risk factors and allow us to simulate the process to obtain information about the joint time paths for the variables including the mean value and the extreme values. With this information, we can obtain the shock scenarios for each risk factor

[8] Many of the individual positions – in particular MBSs, CMOs, and debt securities – have their terms and conditions identified by a CUSIP key. For such CUSIP-identified positions, we use an effective date of July 31, 2005 as well. July 2005 is used as an initial benchmark date for the portfolio in this study and us it only for illustration. We intend to update the date for the benchmark portfolio for further testing.

at a given level of confidence. As a result, it is crucial that our statistical model provide robust forecasts.

A. Modeling Considerations

To generate joint shock scenarios, we must first estimate the dependence structure for the risk factors in a multivariate system. Given no *a priori* structural economic relationships among the risk factors, we chose to model the system using a vector autoregression (VAR) framework. The VAR framework was proposed by Sims (1980) as a method for estimating dynamic economic relationships in a way that avoids imposing "incredible identification restrictions" on the model. Indeed, we impose no restrictions on the system except the risk factors included, which are suggested by the literature, and the lag length, which is estimated statistically. Specifically, the equation for a typical unrestricted VAR is written as

$$\tilde{w}_t = \sum_{i=1}^{p} \alpha_i \tilde{w}_{t-i} + \theta g_t + \eta_t \tag{1}$$

where α_i is a $d \times d$ coefficient matrix, p is the number of lags for the vector of dependent variables $\tilde{w}_t$, θ is a coefficient matrix, g_t is a matrix of exogenous variables, and η_t is a vector of residuals. Since the right-hand side of this equation includes only strictly exogenous (lagged) variables, it can be estimated using ordinary least squares (OLS). We included two exogenous variables in our specification to account for the mean reversion properties of *RST* and *RLT*.[9] Finally, we used the logarithm of the *RLT*, *RST*, and *VOL* series (called *LRLT*, *LRST*, and *LVOL*, respectively) to restrict them to take on only positive values and divided *OAS* by 100 to scale the variable for GARCH estimation.

While the VAR framework is a useful method in cases where a reliable structural model is unavailable, properties of the data may require modifications to this simple model specification. Before we discuss our tests in detail, we provide a broad overview of our approach

[9] We derived the mean-reversion term using an Ornstein-Uhlenbeck process. Specifically, we estimate a two-stage, discrete-time version of the Ornstein-Uhlenbeck process, $dR_t = \beta(R_t - \alpha)dt + \gamma dW_t$. In the first stage, we estimate a discrete-time version of the Ornstein-Uhlenbeck equation. From this equation, we obtain an estimate of the mean of the process (α). We then take this value as fixed and subtract the actual daily short- or long-term interest rate to create a "mean reversion series." These series equal the differences between the estimated mean and the actual interest rates, where a positive number indicates that the actual interest rate is above its mean value. We then use these new series as exogenous variables in our model.

to modeling the DGP. Given this roadmap, we elaborate on our testing procedures, the results, and the choices we make in the modeling process.

The issues of stationarity, cointegration, ARCH, and the appropriate conditional distribution play key roles in determining the appropriate factor model specification. Indeed, we find that the risk factor data are non-stationary, suggesting that a model in first-differences may be appropriate. However, using a model in first differences may result in a loss of information if there exist long run relationships, known as cointegrating relationships, in the data. We tested our model for cointegrating relationships and identified two. Given these cointegrating relationships, we re-specified the VAR model as a VEC model. The VEC specification is written

$$\Delta\widetilde{w}_t = \sum_{i=1}^{p} \beta_i \Delta\widetilde{w}_{t-i} + \chi(\delta\widetilde{w}_{t-1}) + \theta g_t + \eta_t \qquad (2)$$

where $\Delta\widetilde{w}_t$ represents the first-difference of $\widetilde{w}_t$, β_i is a $d\times d$ matrix of coefficients, χ is a $d\times 1$ vector of coefficients that can be interpreted as the speed of adjustments to long run equilibrium, δ is a $1\times d$ vector of normalized coefficients known as the cointegrating vector, θ is a coefficient matrix, g_t is a matrix of exogenous variables, and η_t is a vector of residuals. Finally, we analyzed the errors from the VEC model and found significant evidence of ARCH errors and fat-tails in the conditional distributions. As a result, we modeled the error process as a *t*-GARCH(1,1) process of the following form

$$H_t = \kappa + \Lambda H_{t-1}\Lambda' + \Gamma\eta_{t-1}\eta'_{t-1}\Gamma', \quad \text{with } \eta_t \sim t(0, H_t) \qquad (3)$$

where κ, Λ, and Γ are $d\times d$ matrices of coefficients, and H_t is the conditional variance of η_t. Thus, our final model combines (2) and (3).

We perform our estimation routine in two steps. The first step is to estimate the VEC model using EViews®. The second step is to use the residuals from the VEC model to estimate a GARCH(1,1) model using a program written in Matlab®. Combining the results from these two stages, we obtain a GARCH-VEC model that we use to simulate the joint time paths for the risk factors. We discuss the data issues and their effects on our model specification in more detail below.

B. Data Analysis and Testing

Given the data and the modeling considerations discussed above, we describe in detail the tests we conducted and our conclusions regarding an appropriate model specification for the DGP.

1. Stationarity

A weakly stationary series is one where its unconditional mean is constant and its autocovariance function depends only on the lag.[10] We test for stationarity for two reasons. First, if a series follows a trend (e.g., linear or quadratic) and we difference the data, we have over-differenced and introduced spurious autocorrelation. Instead, we should have modeled the appropriate trend. Similarly, if a series is difference stationary and we fit a trend, we have not removed the stochastic trend, but have introduced spurious autocorrelation. In this case, the data should have been differenced. Second, the test statistics for the regression coefficients in the presence of a unit root do not follow the typical t or F distributions.[11] Rather, they follow a non-standard Dickey-Fuller type of distribution, whose critical values must be computed numerically. Thus, each test statistic would need to be compared to a critical value obtained through time-consuming numerical methods.

We initially conduct two tests for stationarity: the Phillips-Perron (1988) test and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS, 1992) test. We chose the Phillips-Perron and KPSS tests because they allow for a variety of serial correlation and heteroskedasticity patterns. Since our data exhibit both serial correlation and heteroskedasticity, these tests are appropriate.

We perform the Phillips-Perron tests with an intercept assumed in the DGP for each series. For the *RST* and *RLT*, we fail to reject the null hypothesis that there is a unit root in the

[10] An autocovariance function measures the covariance between the data at time t and the data at time $t-s$. More formally, the autocovariance is $E(y_t - \mu)(y_{t-s} - \mu) = \gamma_s$ for all t and any choice of s. See Hamilton (1994), p. 45.

[11] If a series has a unit root, it is said to be integrated of order one, or I(1). In general, a time series is integrated of order g – written I(g) – if the gth difference of the time series is stationary. A stationary time series is I(0). Thus, the first difference of a series that is I(1) is stationary, the second difference of an I(2) series is stationary, and so on. By first-differencing I(1) data, we impose a unit root coefficient on the model and transform the data into a stationary series.

series.[12] For the *LAS*, *OAS*, and *BAS* series, we also perform the Phillips-Perron test without an intercept under the hypothesis that the DGP for these series is a random walk. We fail to reject the null hypothesis that there is a unit root for each series. Table 4 contains the test statistics for the Phillips-Perron tests on each series.

It is well known that the standard unit root tests, including the Phillips-Perron test, fail to reject the null hypothesis of a unit root for many data series. It is also well known that these tests have low power against stationary alternatives with roots close to one. For example, Dejong, Nankervis, Savin, and Whiteman (1992) used the Dickey-Fuller test and could not reject the unit root hypothesis for a number of time series. However, they also found that they could not reject the null hypothesis that the autoregressive term was 0.85. To address the low power of the unit root test, we also conducted the KPSS test. The KPSS test evaluates the null hypothesis that the data are stationary against the alternative that there is a unit root, and provides a parameterization that is valid under both the null and alternative hypotheses. For each data series, we are able to reject the null hypothesis of stationarity. Table 4 contains the test statistics for the KPSS test on each series.

Based on the results of our unit root tests, we conclude that all of the series should be differenced to transform them to stationary processes. Our tests of the differenced series indicate that no further differencing is required. Figure 3 contains plots of each of the first-differenced series.

2. Cointegration

Based on the results of our unit root tests, we concluded that we should use first-differenced data in our VAR model. However, it is well known that differencing data series may result in a loss of information about long-run relationships. That is, while a model in differences is appropriate for capturing short-run dynamics, potentially useful information on longer run relationships (such as equilibrium conditions) may be lost in the process of differencing. Granger (1983) and Engle and Granger (1987) developed the theory of cointegration, which captures both short-run dynamics and long-run relationships.

[12] We applied the Phillips-Perron test using a Bartlett kernel and allowed EVIEWS® to choose the Newey-West bandwidth selection.

Cointegration is defined as follows. Suppose two series y_t and x_t are both I(1). Then y_t and x_t are said to be cointegrated if there exists a linear combination of the two series that is I(0). The cointegrating relationship is $y_t - \beta x_t$, where the coefficient β is called the cointegrating vector. The cointegrating relationship can be interpreted as the long-run relationship between two variables, though it need not have any economically meaningful interpretation. For example, when estimating an open-economy macroeconomic model in first-differences, one might expect to find cointegration in the form of a purchasing power parity relationship.

The first step in testing for cointegration is to determine the appropriate number of lags of the dependent variables in first-differences in an unrestricted VAR. We used both the Schwarz Criterion (SC) and Akaike's Information Criterion (AIC) to determine the number of lags for our system. The results from the SC and AIC suggest that two and six lags, respectively, are appropriate. From a theoretical viewpoint, either two or six lags provides a plausible lag structure for a model in first-differences. That is, there is no *a priori* reason to conclude that daily values for the risk factors are correlated over one lag structure or the other (i.e., two days or six days). Examining the results from VARs with 2 and 6 lags, we conclude that the shorter lag structure was likely to exclude relevant information about the dynamic behavior of the DGP. Thus, we chose to use six lags in our specification.[13]

Given an appropriate lag structure for the VAR, we perform Johansen's (1988, 1991) trace and maximum eigenvalue tests to determine the number of cointegrating relationships. The null hypothesis is that there are no more than h cointegrating vectors in a system with d equations. We test the null hypothesis sequentially for all $h < d$.[14] We perform the Johansen cointegration tests on our system using EVIEWS® and assume there are two lags in the VAR. The form of the test equation is:

$$\Delta \tilde{w}_t = \Pi \tilde{w}_{t-6} + \sum_{i=1}^{5} \Gamma_i \Delta \tilde{w}_{t-i} + Bx_t + \varepsilon_t \, . \tag{4}$$

[13] Researchers have found conflicting results concerning the bias of cointegration tests from using too many lags. Haug (1996) found that including too many lags may actually improve the performance of the cointegration test. The results of our cointegration tests are robust to changing the number of lags from two to six.

[14] If h=d, then the system is stable, or I(0), in levels and should not be estimated in first-differences. If h=0, then there is no evidence of any cointegrating vectors. For a detailed discussion of the Johansen tests, see Banerjee, Dolado, Galbraith, and Hendry (1993).

If the coefficient matrix Π has reduced rank ($h < 6$), then there exist *h* cointegrating relationships. The Johansen test requires estimating the Π matrix using an unrestricted VAR and testing the restrictions implied by the reduced rank.

The first test proposed by Johansen is the trace test. For our model, the test proceeds sequentially by testing the null hypothesis of *h* cointegrating relationships against the alternative of six cointegrating relationships beginning with $h = 0$ and proceeding through $h = 5$.[15] The trace test statistic is:

$$LR_{Trace}(h \mid 6) = -T \sum_{i=h+1}^{6} \ln(1 - \lambda_i), \qquad (5)$$

where λ_i is the i^{th} largest eigenvalue of the Π matrix. The trace statistic is then compared to nonstandard critical values from MacKinnon, Haug, and Michelis (1999) as reported by EVIEWS®.

The second Johansen test is the maximum eigenvalue test. The null hypothesis for this test is that there are *h* cointegrating relationships and the alternative hypothesis is that there are *h+1* relationships. The test statistic is:

$$LR_{Max}(h \mid h+1) = T \ln(1 - \lambda_{h+1}).\,, \qquad (6)$$

We also compare this test statistic to the critical values reported in EVIEWS®. Both the trace and maximum eigenvalue tests indicate that there are no more than three cointegrating relationships among the variables.[16]

Based on the results of the Johansen tests, we specify a VEC model with two cointegrating vectors. We estimate this model in EVIEWS® and obtain coefficient estimates for the cointegrating vector, the adjustment coefficient, and the short-run dynamics. Before we discuss the estimation results, we provide our analysis of the ARCH effects in the residuals of the VEC model.

[15] If *h*=6, then none of the series have unit roots and the equation can be estimated in levels rather than first-differences.

[16] The Johansen tests require an assumption about whether there are any deterministic trends in the data or in the cointegrating relationship. While we do not expect there to be trends in the first differences of the model, but we had no *a priori* assumption regarding whether the cointegrating relationship would have a constant. As a result, we performed the tests both with and without a constant in the cointegrating relationship and found the results to be robust to these different assumptions.

3. ARCH

Failing to appropriately model any ARCH processes would not bias our coefficient estimates, but would result in a loss of efficiency and affect the confidence intervals for our forecasts. However, in our application, the dispersion matrix plays a critical role in defining extreme scenarios. Moreover, validation of our model requires modeling the clustering of periods of volatility and quiescence. Thus, we carefully test for and model the ARCH processes in the risk factors.

Given our VEC specification with a sufficient number of lagged dependent variables to eliminate serial correlation, we first investigate whether the residuals exhibit heteroskedasticity. Specifically, we use EVIEWS® to perform a multivariate extension of White's (1980) test as discussed by Kelejian (1982) and Doornik (1995). The test involves regressing each cross product of the residuals on the cross products of the regressors and tests the joint significance of the regression. The null hypothesis is that there is no heteroskedasticity. Based on our test, we identified statistically significant ARCH effects in our system.

To model these ARCH effects, we use a GARCH model. The GARCH(*p,q*) process models the variance as a function of *p* lags of the squared residuals and *q* lags of the variance. In practice, researchers typically find that a low-order GARCH specification appropriately models the ARCH effects. Our research on the individual series confirms that a GARCH(1,1) model is likely to be appropriate for our multivariate model.

Our descriptive statistics suggest the series are non-normal. We formally test for normality of the residuals from our VEC model. Specifically, we estimate the GARCH model under two joint conditional distributions for the innovations: the normal (Gaussian), and *t* distributions.[17] We estimate our GARCH models under these two distributions and test whether we can reject the restriction imposed on the degrees of freedom parameter in the *t* distribution. With respect to the conditional distributions, we find that the Student's-*t* distribution is more appropriate than the normal distribution, indicating that the series are drawn from distributions that exhibit fat tails.

[17] The Student's-*t* distribution approaches the normal distribution as its estimated degrees of freedom parameter approaches infinity.

C. Model Estimation and Evaluation

As discussed above, we used a two-stage process to estimate the GARCH-VEC model. The first stage is to estimate the VEC model, and the second stage is to use the residuals from the VEC model to estimate the parameters of the GARCH(1,1) process. We discuss the results of each of these stages below.

1. Vector Error Correction Model

We estimated the VEC model using EVIEWS®. We have two cointegrating relationships. Although we have not imposed a structured economic model, these cointegrating relationships reveal long run economic relationships. The first cointegrating relationship reveals that during our sample, the mortgage spread increased when volatility was high, and decreased when the option-adjusted spread and long-term interest rate were high. This could be interpreted as mortgage lenders requiring a risk premium when interest rates are volatile, and absorbing some of the spread (costs and risks) when long-run rates increase. The second relationship reveals that the LIBOR-Agency spread increased with the level of short-term interest rates (and marginally with the option-adjusted spread), but decreased with volatility. Based on our data, it appears that this may reflect that the market appropriately required a risk premium prior to the collapse of the technology stocks around 2000.

The R^2 for the risk factor equations in first differences are:

Equation	R^2
ΔBAS	0.07
ΔLAS	0.15
ΔOAS	0.05
ΔRLT	0.05
ΔRST	0.19
ΔVOL	0.04

An F test of the null hypothesis that all the coefficients in each equation equal zero is rejected at the 1 percent significance level for each equation. Thus, all of the equations provide statistically significant explanatory power for the dynamic process.

Figure 4 contains the residual plots for the series. In each case, the VEC model appears to have eliminated any trends. The presence of substantial ARCH effects is apparent from the residual plots.

Figure 5 contains impulse response plots for each series. Each individual graph displays the response of one variable to a one standard-deviation shock to one of the six variables (including itself) over 250 days. For example, impulse response graphs for *BAS* shows approximately zero effects from *LAS*, *RLT*, *RST*, and *VOL* shocks.[18] The *BAS* response to the *OAS* is positive and persistent. The other plots can be interpreted similarly. In particular, we note the sustained level and persistence of *VOL* shocks, confirming our use of a GARCH model. Our review of the impulse response plots suggests that the effects of shocks stabilize over time, confirming the stability of our estimated model.

2. GARCH

We use the residuals from the VEC estimation to estimate a GARCH(1,1) model.[19] The multivariate GARCH model allows us to estimate not only the conditional variances of the risk factors, but also the conditional covariances between them. Thus, our framework allows for interactions among the volatilities of the risk factors. This is particularly important for our analysis because we will need the variance-covariance matrix to generate scenarios.

In particular, we estimate our GARCH(1,1) model using the BEKK parameterization developed by Engle and Kroner (1995). One advantage of using the BEKK parameterization is that it ensures a positive definite variance-covariance matrix. In addition, by imposing restrictions across and within equations, the BEKK parameterization economizes on the number of estimated parameters without a substantial loss in generality.[20] The BEKK model representation is:

$$H_t = \kappa\kappa' + \Lambda H_{t-1}\Lambda' + \Gamma\eta_{t-1}\eta_{t-1}'\Gamma' \qquad (4)$$

The κ, Λ, and Γ matrices are $n \times n$ coefficient matrices, H_t is the conditional variance-covariance matrix at time t, H_{t-1} is its one-period lag, and η_{t-1} is the residual from the previous period. The

[18] EVIEWS® does not produce confidence bands around impulse responses for VEC models.

[19] For our estimation routine, we programmed the necessary functions in MATLAB®. Many of our programs utilized Kevin Sheppard's code for estimating GARCH models, Sheppard (2008).

[20] See Engle and Kroner (1995) for more detail on the BEKK model.

$\kappa\kappa'$ matrix is symmetric and positive definite. In addition, the remaining terms are in quadratic form, ensuring that the H_t matrix is positive definite. In the context of an autoregressive-moving average model, the $\Lambda H_{t-1}\Lambda'$ term is akin to a moving average term, while the $\Gamma\eta_{t-1}\eta'_{t-1}\Gamma'$ term is like an autoregressive term.[21] Thus, in general the $\Lambda H_{t-1}\Lambda'$ matrix describes the persistence of volatility and the $\Gamma\eta_{t-1}\eta'_{t-1}\Gamma'$ term describes the clustering of volatility.

Our estimation results confirm that there are significant ARCH effects. In addition, we find substantial evidence of volatility spillovers between the risk factors. These volatility spillovers could not be captured in univariate models. Thus, a multivariate framework is important for capturing not only the dynamic relationships between the risk factors, but also the influence of volatility in one risk factor on volatility in another.

To determine whether a normal or *t* distribution was most appropriate, we estimate the model under both assumptions. Then, since the *t* distribution encompasses the normal distribution, we use a likelihood ratio test to test the null of a normal distribution against the alternative of a *t* distribution. We strongly reject the null hypothesis that the data follow a normal distribution. This result is not surprising given that our parameter estimate for the degrees of freedom equals 4.8.

3. Shock Simulation

Given the estimates from our models, we combined them to create a single GARCH-VEC model that we use for simulating possible outcomes. For the simulation, we generate a 6×250 matrix of shocks which represents simultaneous shocks to each of the six risk factors over the course of 250 business days (one year). These shocks are drawn randomly from a *t* distribution with zero mean and conditional standard deviation for shock *i* equal to the simulated value of H_{it}. We perform 10,000 Monte Carlo trials using this simulation model to estimate the expected value and the 0.001 and 0.999 percentiles for each of the series. The expected value for each risk factor one year in the future is equal to the mean value of the risk factor on the 250th day of the forecast. The 0.001 and 0.999 percentiles are the 10th lowest and 9990th highest values in the simulation for each risk factor. We use these values to construct our shock scenarios.

[21] See Hamilton (1994), pp. 665-6.

IV. Portfolio Valuation Examples

A central objective of this research is to learn whether and how the choice of shock scenarios matters to the bottom-line value of the bank. It is therefore important to consider the impact of the generated shock scenarios on the financial status of a representative institution. In this section we analyze the value of a simplified bank exposed to the sets of exposures generated as described above.

Due to extensive hedging of known risk exposures, the residual risks faced by a bank can be highly non-linear. This is especially true of the FHLBs, which typically micro-hedge all new business. The bulk of these risks is linear interest-rate risk (i.e., duration risk), much of which occurs in the advance portfolio. Such linear exposures are straightforward to hedge, largely by match-funding new business. Also important (in terms of notional exposures), however, are the nonlinear prepayment exposures generated by mortgages, MBSs, and CMOs. Prepayable exposures are typically hedged with callable debt and/or callable swaps. With a strategy of near-universal micro-hedging, what remains are the minor unhedged exposures, the basis risk resulting from imperfect hedges, and credit and operational risks.[22]

A. Multivariate shocks

Our multivariate forecasts are jointly normal or *t*-distributed and, therefore, elliptical.[23] To select scenarios from the multivariate forecast distribution, we apply a methodology developed by Flood and Korenko (2008). This approach selects scenarios systematically, at approximately evenly distributed points on the surface of a particular isoprobability ellipsoid, defined by a shock probability analogous to a value-at-risk threshold. The systematic approach avoids the limitation common to many practical scenario-based methodologies, namely that the set of scenarios chosen is, in effect, arbitrary. The methodology exploits certain fundamental properties of multivariate elliptical distributions including the multivariate *t*.

Scenario-based methodologies typically begin by isolating a small number of exogenous

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[22] Credit risk exposures are typically small for the FHLBanks, due to extensive collateralization of assets. In any case, credit and operational risks are outside the scope of this paper.

[23] We performed statistical tests on the skewness and kurtosis of the forecast values. We strongly failed to reject the null hypothesis that the skewness and kurtosis of the distribution are statistically different from a t-distribution with 6 degrees of freedom. See McNeil, Frey, and Embrechts (2005), § 3.3, for a good overview of the properties of elliptical distributions.

stochastic risk factors and selecting from the universe of possible values for those risk factors a subset of events where those risk factors take on extreme values. Given a set of scenarios with extreme values for the risk factors, analysts evaluate the properties of the portfolio (e.g., security values, credit losses, risk-factor sensitivities, etc.) as functions of those extreme risk factors. One advantage of this approach is that the stochastic characteristics of exogenous risk factors are typically easier to analyze than the characteristics of the securities themselves, due to the non-linear nature of many securities, including those with contingent payoffs or embedded option clauses. In many cases, the need for formal risk analysis increases with the amount of non-linearity structured into a security, since users are less likely to have well developed intuitions for the behavior of more complex securities.

The Flood and Korenko methodology measures risk exposures by sampling the universe of possible extreme events in an efficient and comprehensive way. First, we identify a probability threshold that defines the margin of an extreme event as those *combinations* of simultaneous outcomes for the risk factors that occur at a specified quantile, α, of the probability distribution for the risk factors. We chose the 99.9 percent quantile for our present analysis. If the risk factors obey a multivariate elliptical distribution, then, by definition, the marginal set of extreme events – those on the α^{th} isoprobability contour – form an ellipsoid in the space of possible risk-factor outcomes. Given this isoprobability ellipsoid, we choose scenarios that form a mesh of points that span all dimensions of the risk-factor space and are regularly spaced on the surface of the ellipsoid, in a sense to be defined below. Such a mesh of points can be chosen systematically, thus avoiding the pitfall of "blind spots" due to conscious or unconscious selection bias when sampling the state space.

These scenarios have two particularly attractive properties. First, they are representative of all possible combinations of the risk factors. Thus, they encompass not only the combinations of risk factors that may have been observed historically, but also those combinations of risk factors that have not yet been observed. Second, all of the scenarios are generated based on the properties of the distributions of the risk factors. To the extent that the joint distribution has been estimated accurately, the scenarios are necessarily consistent with economic reality. That is, even scenarios that have never been observed historically are consistent with the behavior of the risk factors.

B. Response to shocks

Although the main goal of this paper is to present a systematic process for generating plausible shock scenarios, the ultimate impact of those shock scenarios is a question of obvious interest.[24] We subjected the hypothetical institution described in Table 3 to a battery of shocks generated systematically as described above. Specifically, we analyzed the full set of positions for each of the shock scenarios including a base-case "shock," meaning no change in any of the six risk factors, and the 64 multivariate shocks generated as described above.
There are thus a total of 65 shock scenarios to compare.

Although the portfolio analytics tool produces a variety of diagnostic results, we focus the post-shock market value of equity (MVE) as the metric most relevant for issues of risk-based capital. We calculate MVE as the sum of the post-shock model values (or "prices") of the asset portfolios, less the liability portfolios, plus the net swap values, all as a percentage of the post-shock asset value:

$$MVE = \frac{\sum Assets - \sum Debt + \sum Swaps_{net}}{\sum Assets} \times 100$$

The pre-shock MVE for the hypothetical bank is 5.55%. The largest exposures affecting the individual portfolios, of course, have been hedged out in the aggregate: MVE tracks the net residual exposure to the risk factors.

Our analysis of the post-shock MVEs for all 65 scenarios reveals that most of the shocks have a negative impact on the bank.[25] This suggests that the bank's residual exposures may be dominated by second-order effects – i.e., negative convexity. Thus, the bank is typically punished both by large up moves and large down moves.

To test this more closely, we fit a full second-order response surface to the MVE data. That is, we regress the post-shock MVE on the six risk factors in a least-squares regression. On the right-hand side, we include both linear and quadratic versions of each of the six factors, as well as cross-product terms for each combination of different risk factors:

$$MVE_i = \sum_{d=1}^{6} z_{id} + \sum_{d=1}^{6} z_{id}^2 + \sum_{d=1}^{6} \sum_{c=d+1}^{6} z_{id} z_{ic} + \varepsilon_i$$

[24] Caveat: the shock-response results presented in this section are tentative, contingent on verification and testing of our model configuration.

[25] Indeed, under some scenarios we are able to generate negative MVEs.

Including the intercept, there are 27 explanatory variables and 64 observations. We apply the White correction for heteroskedasticity. The results appear in Table 5.

The adjusted R^2 for the regression is very high, 79 percent, which is unsurprising, given that we have, by construction, used the universe of exogenous risk factors on the right-hand side. The terms involving the products of the *OAS* and mortgage basis (*BAS*) and the short-term interest rate (*RST*) and *BAS* are significant at the 10 percent and 5 percent levels, respectively. This is consistent with well known difficulties in hedging mortgage risk. The coefficient on the *LAS* is positive and significant, possibly reflecting increased profitability when there is a "flight to quality." The *LAS* is also important in that the FHLB members may enjoy lower advance rates or higher dividends when the FHLB is able to borrow at lower rates. Also significant at the 5 percent level are the three terms reflecting the interaction between the *RST* and the *BAS*, *LAS*, and *OAS*. The significance of the second-order terms is consistent with the hypothesis that the residual exposure is non-linear and includes substantial negative convexity.

V. Conclusions

We analyze the joint time-series properties of a set of six stochastic risk factors deemed important to the residual risk exposures of the FHLBs. After fitting a VEC model with a GARCH(1,1) error process to the series, we generate joint distribution for the 250-day forecast. We then apply an algorithm to systematically select shocks that are representative of the six-dimensional ellipsoid characterized by the joint forecast distribution. Finally, we apply these shocks to a hypothetical bank modeled after a generic FHLB portfolio strategy. This application demonstrates the feasibility of our scenario-generating mechanism. We find that the residual exposures of our hypothetical institution are non-linear, with evidence of negative convexity.

VI. References

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Table 1: Raw Data

Name	Source	Identifier	Dates
3-mo. LIBOR-FHLB swap spread	Bloomberg	SRLB3MTH	1994/04/04-2008/05/05
3-mo. T-Bill rate	FRB St. Louis	DTB3	1994/04/04-2008/05/05
10-year T-Bond rate	FRB St. Louis	DGS10	1994/04/04-2008/05/05
TBA par mortgage rate	Bloomberg	MTGEFNCL	1994/04/04-2008/05/05
Mortgage OAS over par	Lehman Live	FNMA 30yr CC 2005	1994/04/04-2008/05/05
Implied volatility (swaptions)	Lehman Live	USD 3M 1Y BP Vol	1994/04/04-2008/05/05

Table 2a: Descriptive Statistics for Scenario Data in Levels

	RST	***RLT***	***VOL***	***LAS***	***OAS***	***BAS***
Mean	3.96	5.30	90.64	4.28	46.91	1.67
Median	4.70	5.12	86.80	5.13	41.93	1.59
Maximum	6.42	8.04	159.90	6.74	154.96	2.76
Minimum	0.61	3.13	42.00	0.91	7.31	1.31

Table 2b: Descriptive Statistics for Scenario Data in Differences

	BAS	*LAS*	*OAS*	*RLT*	*RST*	*VOL*
Mean	0.000181	-0.00054	0.003576	-0.00094	-0.00065	0.002569
Median	0	0	0.005417	0	0	0
Maximum	0.204	0.26	18.50159	0.34	0.61	17.7
Minimum	-0.286	-0.61	-36.1067	-0.27	-0.64	-15.4
Std. Dev.	0.025885	0.034629	2.689392	0.059373	0.057522	2.852689
Skewness	-0.10865	-5.33683	-0.60375	0.272518	-0.63591	0.516582
Kurtosis	13.48619	76.81164	17.30912	4.790185	30.51815	9.868415
Jarque-Bera	16056.5	811832.6	30097.92	511.1208	110762.8	7041.402
Probability	0	0	0	0	0	0
Observations	3503	3503	3503	3503	3503	3503

Table 3: Portfolio Strategy Assumptions

A/L/O	Pct.	Description
A	59%	FHLB advances
A	13%	Investments: Money-market
A	12%	Whole-loan mortgages: 15-30 year
A	16%	Mortgage securities: Whole-loan MBS and CMO
	100%	**Total assets**
L	24%	Debt: Discount notes
L	4 %	Debt: Floating-rate bonds
L	26%	Debt: Callable bonds
L	31%	Debt: Non-callable bonds
L	8%	Debt: Other
	93%	**Total debt**
OBS	–	Interest-rate swaps
	–	**Total derivatives**

Table 4: Results of Unit Root Tests

	Phillips-Perron with intercept (null = unit root)	**KPSS (null = stationary)**	**Phillips-Perron no intercept (null = unit root)**
RST	-0.46842	*2.83205	-
RLT	-1.81715	*5.66289	-
VOL	*-3.63047	*1.40266	-0.89628
LAS	-0.45287	*2.71896	-0.73013
OAS	*-3.14664	*0.60848	-1.02852
BAS	*-2.86901	*0.93086	0.05320
Notes:	* represents statistical significance (rejecting the null hypothesis) at the 5 percent level.		

Table 5: Response of MVE to Risk Factors

Dependent Variable: MVE
Method: Least Squares
Included observations: 64
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.	
C	0.086007	0.224320	0.383411	0.7036	
BAS	-0.004989	0.058486	-0.085311	0.9325	
BAS2	0.012906	0.012579	1.026030	0.3115	
BAS_LAS	0.010034	0.016554	0.606161	0.5481	
BAS_OAS	-0.000376	0.000215	-1.745490	0.0892	*
LAS	0.082376	0.043280	1.903336	0.0648	*
LAS2	0.013475	0.009248	1.457066	0.1535	
LAS_OAS	-0.000145	0.000209	-0.694889	0.4915	
OAS	0.000248	0.000727	0.340969	0.7351	
OAS2	1.47E-06	1.19E-06	1.236519	0.2241	
RLT	-0.006411	0.007638	-0.839265	0.4067	
RLT2	8.77E-05	0.000154	0.568358	0.5732	
RLT_BAS	-2.84E-05	0.002204	-0.012883	0.9898	
RLT_LAS	-0.000538	0.003531	-0.152260	0.8798	
RLT_OAS	5.93E-06	1.76E-05	0.336826	0.7382	
RLT_VOL	2.61E-05	1.93E-05	1.351456	0.1848	
RST	0.037052	0.030671	1.208063	0.2347	
RST2	0.003564	0.004474	0.796460	0.4308	
RST_BAS	-0.022520	0.009639	-2.336379	0.0250	*
RST_LAS	-0.011313	0.003781	-2.992008	0.0049	*
RST_OAS	0.000205	8.57E-05	2.392415	0.0219	*
RST_VOL	6.93E-06	5.83E-06	1.188748	0.2421	
VOL	-0.000283	0.000390	-0.725793	0.4725	
VOL2	1.04E-07	8.90E-08	1.171694	0.2488	
VOL_BAS	2.36E-05	3.66E-05	0.644105	0.5235	
VOL_LAS	4.10E-05	5.48E-05	0.749351	0.4584	
VOL_OAS	-3.92E-07	7.25E-07	-0.540374	0.5922	

R-squared	0.786864	Mean dependent variable	-0.021026
Adjusted R-squared	0.637092	S.D. dependent variable	0.127275
S.E. of regression	0.076673	Akaike info criterion	-2.002749
Sum of squared residuals	0.217514	Schwarz criterion	-1.091970
Log likelihood	91.08795	Hannan-Quinn criterion	-1.643947
F-statistic	5.253761	Durbin-Watson statistic	2.281620
Probability (F-statistic)	0.000003		

Figure 1: Process flow overview

(Primary source data are highlighted in yellow.)

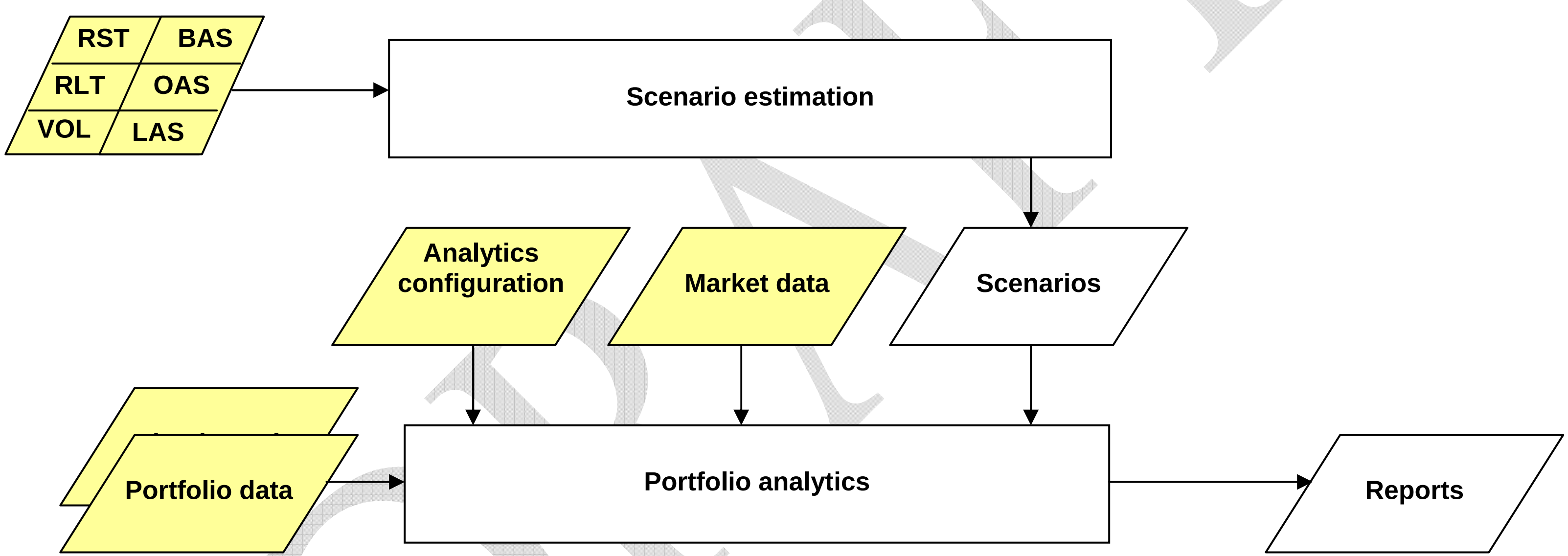

Figure 2: Plots of raw data

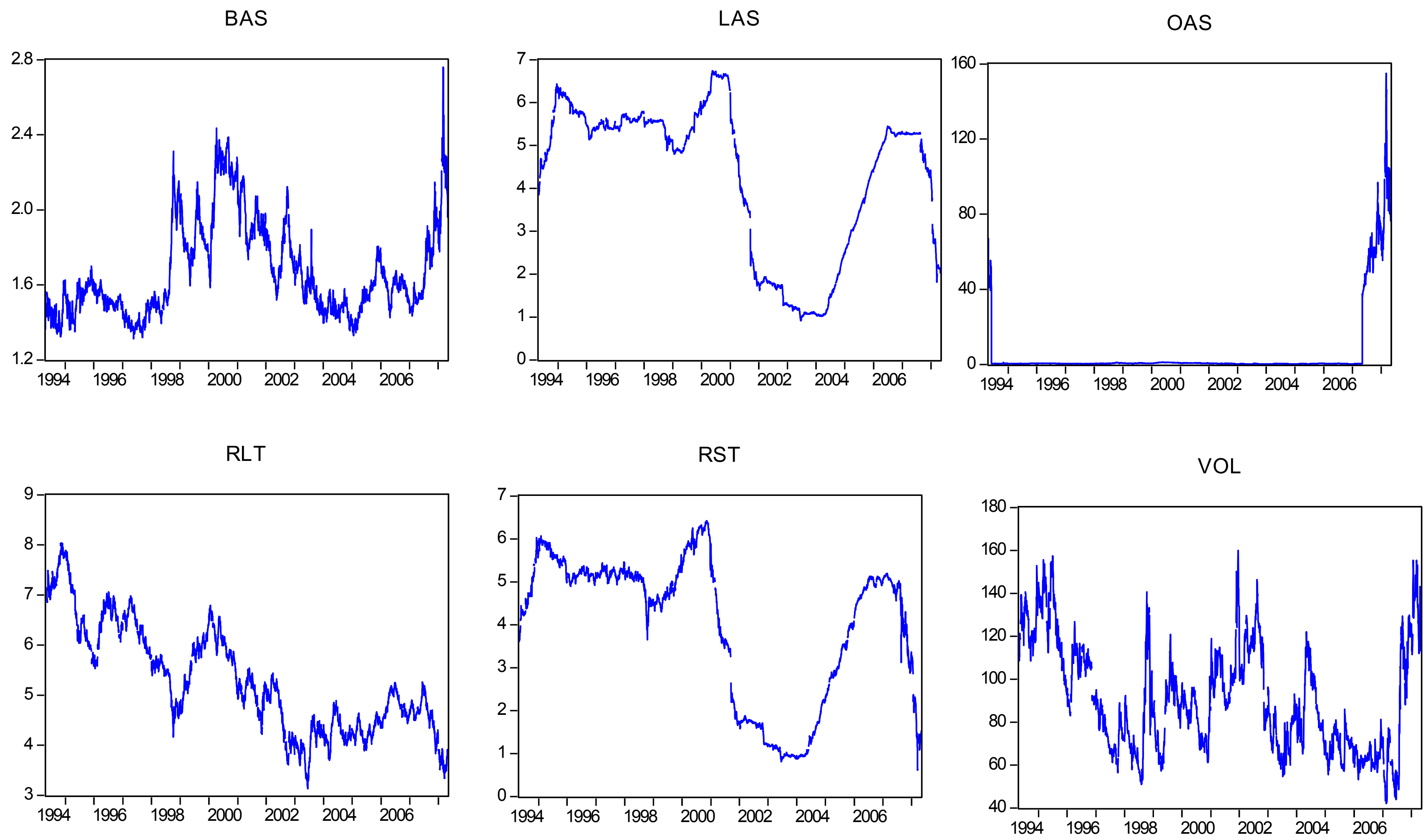

Figure 3: Plots of first-differenced data

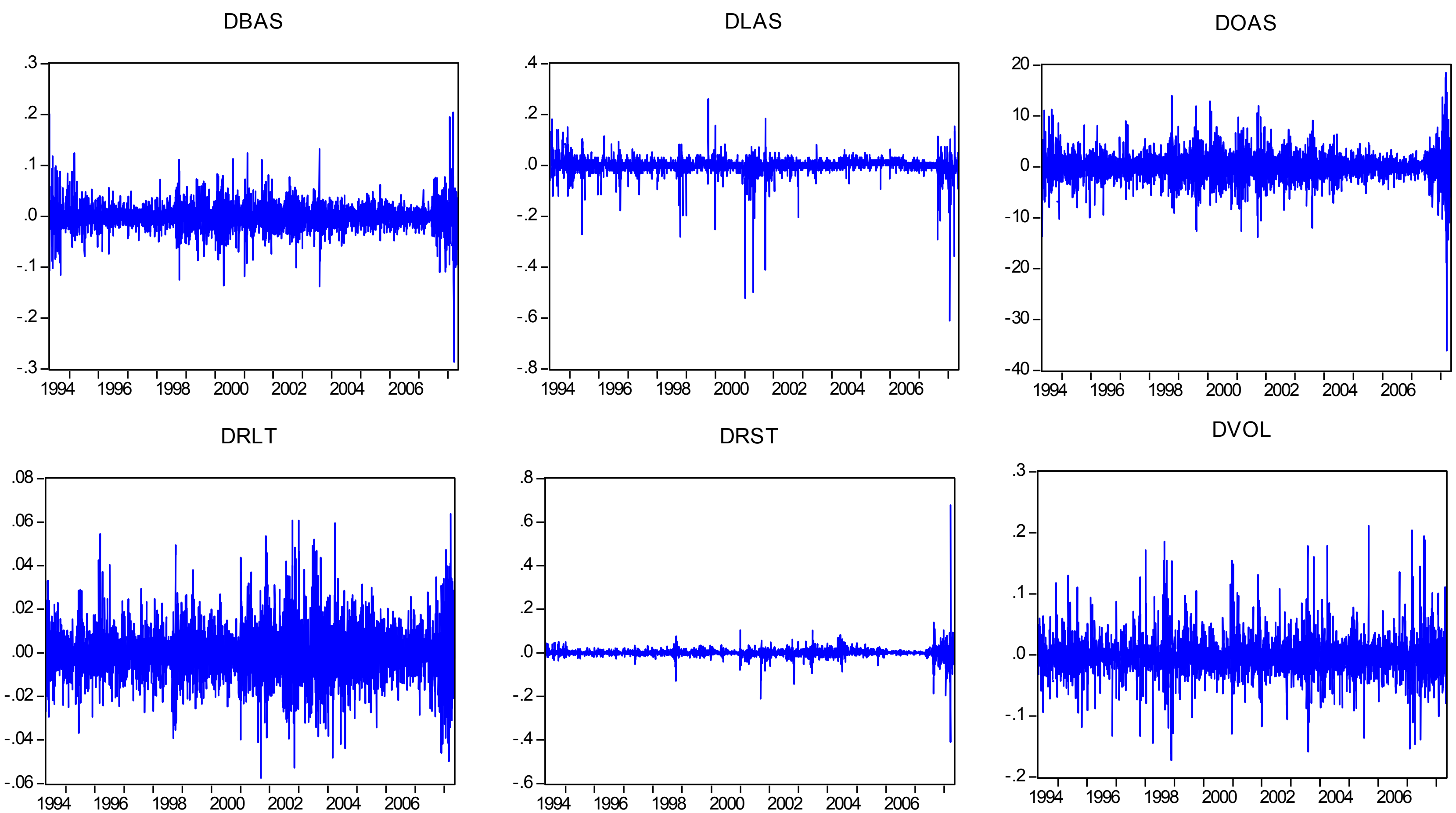

Figure 4: Error-correction model residual plots

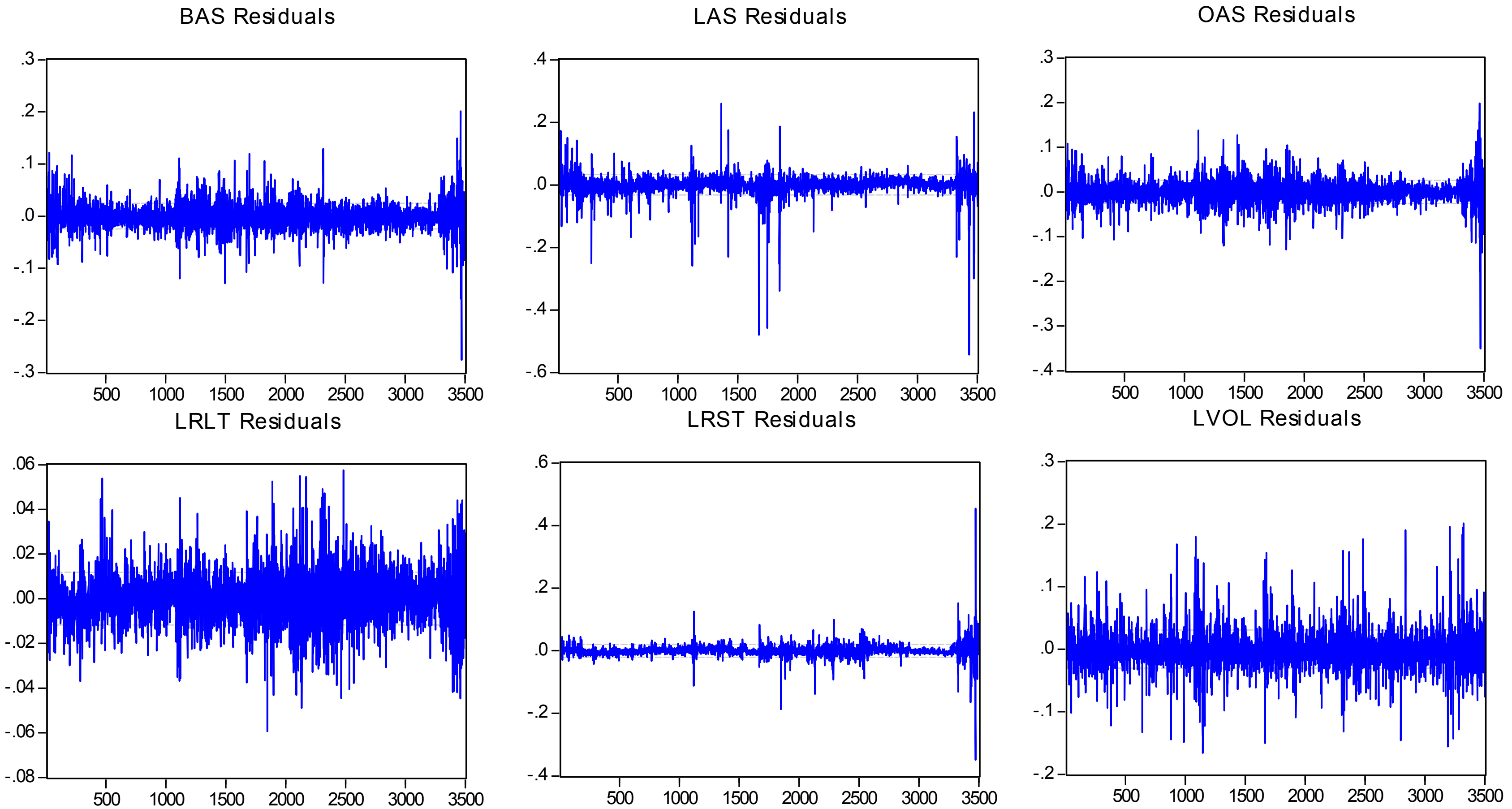

Figure 5: Impulse responses