

Bayesian Inference for Issuer Heterogeneity in Credit Ratings Migration*

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Abstract

Rating transition matrices for corporate bond issuers are often based on fitting a discrete time Markov chain model to homogeneous cohorts. Literature has documented that rating migration matrices can differ considerably depending on the characteristics of the issuers in the pool used for estimation. However, it is also well known in literature that a continuous time Markov chain gives statistically superior estimates of the rating migration process. It remains to verify and quantify the issuer heterogeneity in rating migration behavior using a continuous time Markov chain. We fill this gap in literature. We provide Bayesian estimates to mitigate the problem of data sparsity. Default data, especially when narrowing down to issuers with specific characteristics, can be highly sparse. Using classical estimation tools in such a situation can result in large estimation errors. Hence we adopt Bayesian estimation techniques. We apply them to the Moodys corporate bond default database. Our results indicate strong country and industry effects on the determination of rating migration behavior. Using the CreditRisk+ framework, and a sample credit portfolio, we show that ignoring issuer heterogeneity can give erroneous estimates of Value-at-Risk and a misleading picture of the risk capital. This insight is consistent with some recent findings in literature. Therefore, given the upcoming Basel II implementation, understanding issuer heterogeneity has important policy implications.

1 Introduction

Markov chains (c.f. Norris (1997)), and more specifically, a time-homogenous, discrete-time Markov chain has been extensively used to model the ratings migration process for corporate bonds and bond issuers. Having accepted this model, the actual reported transition probability matrices can vary considerably depending on the actual data and estimation methodology used (c.f. Altman (1998)). Such modelling has often further assumed that the rated entities are homogeneous with respect to their rating migration behavior. Deviation from this added assumption has been the subject of several studies that highlight latent and observable sources of heterogeneity such as the issuer's age, country of domicile, stage in the business cycle etc. (c.f. Frydman and Schuermann (2007), Chava, Stefanescu, and Turnbull (2006), Frydman and Kadam (2004), Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002), Nickell, Perraudin, and Varotto

(2000), Lucas and Lonski (1992), Asquith, Mullins, and Wolff (1989) and research summary reports published by rating agencies such as Moody's KMV, Standard & Poor's and Fitch on their web-sites.) However, it is also well known in literature that a continuous time Markov chain gives statistically superior estimates of the rating migration process. (c.f. Jarrow, Lando, and Turnbull (1997), Lando and Skodeberg (2002) and Christensen, Hansen, and Lando (2004).) It remains to verify and quantify the issuer heterogeneity in rating migration behavior using a continuous time Markov chain. We fill this gap in literature. Such an exercise would be redundant if every observed rating transition matrix of a discrete time Markov chain could result unambiguously from a continuous time Markov chain rating migration process. However, in general, neither existence nor uniqueness of solution to this embedding problem is assured. (c.f. Schonbucher (2003), Bluhm, Overbeck, and Wagner (2002) or Israel, Rosenthal, and Wei (2001).)

In some sense our modelling framework is similar to Frydman and Kadam (2004) and Frydman and Schuermann (2007). Both of these apply continuous time Markov chain based mixture models to ratings data. The discrete time model of Chava, Stefanescu, and Turnbull (2006), explicitly addressing issuer heterogeneity, has a similar motivation. However, all of these use Maximum Likelihood Estimation for model calibration.

We provide Bayesian estimates to mitigate the problem of data sparsity. Default data, especially when narrowing down to issuers with specific characteristics, can be highly sparse. Using classical estimation tools in such a situation can result in large estimation errors. In contrast, Bayesian methods have two major advantages.

The first advantage of Bayesian methods in this context is that of estimation accuracy. Stefanescu, Tunaru, and Turnbull (2007), who also advocate Bayesian methodology for calibrating models for rating transition probabilities using historical data, assert “Model calibration for this type of application is difficult in a classical frequentist estimation framework, because the sparsity of data often leads to unrealistic transition probabilities”. Because of the nature of estimation procedure, we are able to provide estimates for an arbitrary issuer profile even if data on that profile may be a very small part of the sample we use for estimation. For instance the rating evolution for Japanese issuers in the Utility sector can be estimated although this type of issuers comprise only 0.1% of the data. This is made possible by combining the information on Japanese issuer transitions (3% of the sample) and on Utility sector issuer transitions (10% of the sample). The estimation error in doing this using a frequentist approach may be quite large.

The second advantage of Bayesian estimation is the incorporation of expert opinion or subjective beliefs (such as for instance those imposed during stress-tests for models) via prior distributions for rare events (such as rating transitions or defaults). As pointed out by McNeil and Wendin (2007), who also advocate Bayesian estimation for portfolio credit risk applications, this “could, in a sense, allow us to draw stronger conclusions about default risk than is possible from an analysis of empirical defaults alone”. In our empirical analysis, we used highly noninformative priors but given default data sparsity, incorporating prior beliefs is a valuable tool that can be potentially prove quite useful.

A third side-benefit of using Bayesian inference is also that it becomes straightforward to compute the transition or default probability interval estimates which are becoming increasingly popular; see for instance Christensen, Hansen, and Lando (2004). We do not provide such estimates here so as not to distract from our primary focus viz. heterogeneity which can be demonstrated with point estimates.

Our empirical results build upon the work of Nickell, Perraudin, and Varotto (2000), who made a significant contribution to literature by fitting a Probit model to discrete rating data. Their model-based approach allows for each qualifier of interest (e.g. country of domicile), a conditional transition matrix (over a given time period), estimated by conditioning on values taken by that variable (e.g. USA, UK and Japan), having controlled for other sources of variation (e.g. industry type). Their ordered probit model assumes that rating changes when an unobserved, latent measurement falls into disjoint, adjacent intervals. An advantage of this approach is that a common set of parameters for the latent measure is used for each rating state. In our model, the transition parameters depend on the current state of the process, which provides a more flexible model than the ordered-probit at the cost of a significantly larger parameter space. Fortunately, the issuer rating dataset we use is large, and Bayesian inference enables the estimation of a large number of parameters. Furthermore, we explicitly model duration viz. the time spent by an issuer in the current rating before making a transition to the future rating. Modelling the duration explicitly allows us to provide a richer understanding of rating stability. The variability in duration times for the dataset we used is quite high both within and across rating categories. This suggests

that the average stay period in any given rating is not a reliable summary statistic. A key feature of this paper vs. other discrete time Markov chain model based papers (such as Nickell, Perraudin, and Varotto (2000)) is that duration times have a model that captures this large variability.

Given the use of Bayesian techniques in ratings migration context, our work shares some similarities with McNeil and Wendin (2006). Their model, however, is quite different. It is set in discrete time, and not Markov chain based. In particular, it allows for serially correlated unobserved risk factors that affect the rating migrations process. This serial correlation gives joint migration distributions in terms of high dimensional integrals, which are awkward for standard maximum likelihood procedures; Bayesian estimation circumvents this problem.

Our results indicate strong country and industry effects on the determination of rating migration behavior. For instance, issuer default probability shows a clear ordering across countries: UK > Canada > US > EU > Japan. Utility sector issuer ratings are generally more stable and whereas Banking sector issuer ratings are generally less stable; Industrial sector issuer ratings lie somewhere in between. A possible explanation for country heterogeneity is the cross-country variation in bankruptcy codes, corporate governance and accounting standards. A possible explanation for sector heterogeneity is the cross-sector variation in the uncertainty of future revenue streams.

Using the CreditRisk+ framework, and a sample credit portfolio, we show that ignoring issuer heterogeneity can give erroneous estimates of Value-at-Risk and a misleading

picture of the risk capital. This insight is consistent with some recent findings in literature. Using a portfolio composition different from ours, a different dependency model between obligors, different data and different (classical) estimation methods, Hanson, Pesaran, and Schuermann (2007) show that “heterogeneity in the ... probability of default, measured for instance by a credit rating, is of first order importance in affecting the shape of the loss distribution”. Being able to explicitly recognize the heterogeneity in the issuer pool gives us a clearer picture of both Value at Risk and risk capital, both of which depend crucially on the loss distribution. Therefore, given the upcoming Basel II implementation, understanding issuer heterogeneity has important policy implications.

The rest of the paper is organized as follows: Sections 2 and 3 summarize the model and estimation procedure used. Sections 4 and 5 describe the data and empirical results obtained. Section 6 demonstrates some implications for holding risk capital and section 7 presents the conclusion.

2 Model

We model the changes in an issuer’s rating over time as a discrete space, continuous time, stationary Markov process. These Markov processes can be represented by the duration time that the process is in a state and transition probabilities or jump distributions for a transition to a new state. The duration times are independent and exponentially distributed with rate parameters that depend on the issuer’s current rating. At the end of the duration, the rating jumps to a new rating. The jumps and

durations are mutually independent within an issuer.

We index the states by $k = 1, \dots, K + 1$. The states $1, \dots, K - 1$ are ordered such that as the index increases, credit quality deteriorates. State K corresponds to the rating being withdrawn, and state $K + 1$ is default, which is absorbing. We observe the ratings process for a set of issuers where i indexes the issuer for $i = 1, \dots, M$. The observational time period is $a \leq t \leq b$.

During the observation period, issuer i has n_i transitions or changes in its ratings. The j^{th} transition in the rating for issuer i occurs at time $T_{i,j}$ for $j = 1, \dots, n_i$ where $a \leq T_{i,1} < \dots < T_{i,n_i} \leq b$. At time t such that $T_{i,j} \leq t < T_{i,j+1}$ the issuer's rating is in state $s_{i,j}$. The rating then changes at time $T_{i,j+1}$ to $s_{i,j+1} \neq s_{i,j}$. Duration times are defined from the transition times. The duration time for the j^{th} transition for issuer i is: $D_{i,j} = T_{i,j+1} - T_{i,j}$ for $j = 1, \dots, n_i - 1$. Since the observation period is a finite interval, we need to carefully deal with the left and right truncation of the observed rating process. These truncation technicalities are explained in Appendix I.

In discrete space, continuous time, stationary Markov processes, the duration times are mutually independent and exponentially distributed random variables. Suppose the density for duration $D_{i,j}$ is $f(t|y_{i,j}) = \exp(-y_{i,j}) \exp[-\exp(-y_{i,j})t]$ for $t > 0$ with rate parameter $\exp(-y_{i,j})$ and expected value $E(D_{i,j}|y_{i,j}) = \exp(y_{i,j})$. Suppose that the issuer is in state $s = s_{i,j}$ during duration $D_{i,j}$. Our model for the $y_{i,j}$ is:

$$y_{i,j} = x_i' \beta_s + \phi_{i,D} + \epsilon_{i,j} \tag{1}$$

where x_i is a p -vector of covariates for the issuers; β_s is a p -vector of regression coefficients; $\phi_{i,D}$ is a random effect for issuer i ; and $\epsilon_{i,j}$ are error terms. The covariates used for implementation were dummy variables to capture country and industry effects.¹ Both the random effects and the error terms are mutually independent and normally distributed with mean zero. The variance of the error terms depends on the state s : $\text{var}(\epsilon_{i,j}) = \sigma_s^2$. The variance of the random effect is λ_D^2 .

At time $T_{i,j}$ the issuer has a transition from state $r = s_{i,j-1}$ to state $s = s_{i,j}$ where $r \neq s$. The transition probabilities are conditional on the previous state r . If $r = K + 1$ is the absorbing (default) state, then the process ends. We choose to model the probabilities of jumping from r to s as logistic functions.

$$P(r|r, i) = 0 \tag{2}$$

$$P(s|r, i) \propto \exp(z_i' \alpha_{r,s}) \text{ for } s = 1, \dots, K \text{ and } s \neq r \tag{3}$$

$$P(K + 1|r, i) \propto \exp(\phi_{i,A}) \text{ for the absorbing (default) state } K + 1 \tag{4}$$

where z_i is a q -vector of covariates for issuer i ; $\alpha_{r,k}$ is a q -vector of coefficients; $\phi_{i,A}$ is a random effect that measures propensity of the issuer to default.

The covariates used for implementation were dummy variables to capture country and industry effects. The z_i covariates for model implementation were identical to the x_i covariates. This choice is by convenience, and not a restriction imposed by either the model or the estimation method.

¹In a different context, Aretz and Pope (2007) decompose the systematic variation of shocks to firms default probabilities into (a global,) a country and an industry effect and identify strong country effects.

The random effects $\phi_i = (\phi_{i,A}, \phi_{i,D})'$ for issuer i are random samples from a mean-zero, bivariate normal distribution with covariance matrix $\Lambda = \begin{bmatrix} \lambda_A^2 & \lambda_{AD} \\ \lambda_{AD} & \lambda_D^2 \end{bmatrix}$.

Stationary Markov processes can also be compactly represented by their generators. The generator for the rating migration process of issuer i in our framework depends on the value of the covariates, the random effects, and the error terms. In a slight abuse of notation, $\epsilon_{i,s}$ is the error term for the ln-rate model for durations when the issuer is in state s , and $y_{i,s} = x_i' \beta_s + \phi_{i,D} + \epsilon_{i,s}$. The generator for issuer i can now be written as the $K + 1$ by $K + 1$ matrix

$$Q_i(\phi_i, \epsilon_i, x_i, z_i) = \begin{cases} -\exp(-y_{i,j}) & \text{for the } (j, j) \text{ element and } j = 1, \dots, K \\ \exp(-y_{i,j})P(k|j, i) & \text{for the } (j, k) \text{ element where} \\ & j = 1, \dots, K; k = 1, \dots, K + 1; j \neq k \\ 0 & \text{for the } (K + 1, k) \text{ element; } k = 1, \dots, K + 1. \end{cases} \quad (5)$$

3 Estimation methodology

We use Bayesian inference (c.f. Congdon (2001) for a general introduction; Allenby and Lenk (1994) and Allenby and Lenk (1995) discuss panel data specifics) to estimate the proposed model for ratings migration. Bayesian inference is particularly well suited in capturing random effects and parameter heterogeneity in repeated observation studies, such as ours, where there are a large number of issuers and relative few rating transitions for each issuer. For the dataset we used, the percentage of issuers making exactly 1, 2 and 3 transitions in their entire life is roughly 30%, 20% and 10%. Fur-

thermore, the median of the number of transitions made by issuers during their entire lifetimes is 2. The sparsity is likely to be even more pronounced when narrowing the sample to some specific cross section of issuers such as those in a particular industrial sector or country of domicile. In this situation, traditional estimates at the issuer level either do not exist or have large sampling variability.

Bayesian inference automatically shrinks the maximum likelihood estimate (MLE), if it exists, to an aggregate or pooled estimate based on all of the data. The amount of shrinkage depends on a variety of factors, such as the sampling variation of the issuer-specific MLE and the heterogeneity among the issuers. When the issuer-specific MLE does not exist, the Bayes estimate does by incorporating information from all of the issuers. In sparse-data situations, the issuer-specific estimates reflect the aggregate behavior of the data. As more observations are obtained for a particular issuer, the Bayes estimate reflects less on the aggregate behavior and more on the data for the specific issuer.

We used Markov chain Monte Carlo (MCMC) (c.f. Congdon (2001)) to analyze the model. MCMC sequentially generates the subsets of the parameters from the “full conditional” distribution given the data and the other sets of parameters. Except for the generation of the ln-rate parameters $y_{i,j}$, the MCMC uses standard algorithm. We generate $y_{i,j}$ by using the “slice sampling” method of Damien, Wakefield, and Walker (1999). The initial “burn-in” period of our MCMC chains consisted of 100,000 iterations. We then generated another 100,000 iterations for estimation. To conserve memory, we thinned the chained by only using every tenth iterations, for a total of

10,000 iterations to compute posterior means and posterior standard deviations of the parameters. We also computed the generator for various values of the covariates on each of the 10,000 iterations. On each of these iterations, we generated 100 random effects ϕ_i , and computed the generator for each draw of the random effects and error terms. In total, we computed the generator 1,000,000 times for each setting of covariates. We used highly noninformative priors.

4 Data Description

The dataset we use is the entire Moodys corporate bond default database available as of late 2005. Most of the rating transitions in this dataset occur after 1970. The last recorded rating transition is in April 2005. This rich dataset provides us rating histories from 112 countries and 14 industry sectors. The model implementation uses dummy variables for countries and industries as covariates in the duration and transition models. To improve both execution speed and output interpretation, it is desirable to have fewer countries and industry sectors for the model implementation. To this end, we eliminate those countries and industry sectors that have a very small number of rating transitions. In doing so we first merge all countries in the European Union and treat it as one country EU. This leaves us with the countries USA, UK, Japan, Canada and EU. To fit the model, we focus on the following 7 industry sectors : Banking, Utility, Insurance, Transport, Government, Finance and Real Estate Finance, eliminating the remainder which contain a very small fraction of the data. The original data (all countries and sectors) had 27231 rating transitions. After selecting countries

and industries, there were 22983 rating transitions remaining. Therefore, about 15% data was discarded in this process.

In this final dataset majority (over 80%) of the coverage is for US issuers.² Similarly majority (over 75%) of the data relates to the Industrial sector, and of the remainder, substantial parts relate to Banking and Utility sectors. For the rest of the study we focus on these three sectors.

As is customary we grouped the original ratings into eight states: Aaa, Aa, A, Baa, Ba, B, C, D and WR. The ratings are ordered from the highest to the lowest with Aaa being the top ranking, D being the default state and WR denoting the state of rating withdrawal. In general, there are very few rating category transitions per issuer and it is rare for an individual issuer to make more than three transitions in its life. From a cross tabulation of rating it follows that majority of the transitions are to neighboring states, and that there are substantially more downgrades than upgrades. A large proportion (over 20%) of these transitions were to the Withdrawn state. An even larger proportion (over 30%) were to End state. These censored observations were not incorporated in the estimates of transition probabilities. As is customary in literature, a transition probability matrix estimate does not report the transition probability to End state.

Apart from the transitions themselves, a key quantity of interest is the duration of time spent in each state. The censored observations were indeed useful in enriching

²Moody's coverage used to be largely focused on US issuers but in the recent times has become more and more international.

the estimates of duration times. Extracting this additional information from censored observations is facilitated by the fact that we employ a continuous time framework. Computing summary statistics of durations for each of the rating categories indicates that higher rated issuers spend more time in their current rating category before making a transition and that the variability in duration times is more for higher rated issuers.

5 Empirical Results

5.1 Estimates for the standard profile

Of primary interest to us is the generator for the continuous time Markov chain, which can be directly exponentiated to get the one year transition probability matrix. In Table 1 we present the estimated generator for US issuers in the Industrial sector. These issuers make up more than half of our data, and we treat this profile as the standard profile. It is important to note that we estimate the generator using a day as the unit of time, so the diagonal entries of the generator are to be interpreted as exit rates per day (and not per year).

Estimates for issuers from other countries (we focus on UK, Japan, Canada and EU) or other industry sectors (we focus on Industrial, Banking and Public Utility) will differ from the above standard profile estimates due to inherent heterogeneity in the rating migration behavior. The purpose of this study is to quantify and analyze that difference.

5.2 Estimates for other profiles

In the interest of brevity we do not tabulate generators and transition probability matrices for each possible profile. To illustrate the heterogeneity we choose a few prominent country-sector combinations and compare their estimates with those for the US-Industrial issuers. For the sake of illustration, one year transition probability matrices for US issuers in the Industrial, Banking and Utility sectors (these comprise over three fourths of our data) are given in Table 2. They show strong sector effects. For instance, Banking and Utility sector issuers have about 7 – 8% lower chance of default. Diagonal entries can vary drastically (e.g. see B or C) and as do upgrade probabilities (e.g. see BAA, BA). Similarly, comparing one year transition probability matrices (not presented here in the interest of brevity) we find prominent country effects within sectors.

In order to formalize these systematic differences a first approach could be to consider some distance measure between generators (or transition probability matrices) of two issuer profiles and check if this distance is significantly different from zero. In a Bayesian context, the distance measure will have its own posterior probability distribution. Hence a distance significance check could be interpreted as checking if the mean of the distance is significantly larger than the standard deviation of the distance. We found that for any issuer profile (characterized by a country-sector combination) its distance from all other issuer profiles was quite large. Without exception the mean distance was at least thrice the standard deviation of distance, usually much larger. This was true for generators as well as transition probability matrices, and for L1 as

well as L2 measures of distance. Thus our first approach indicates that rating migration behavior is statistically significantly different across the issuer profiles we considered.

In order to further quantify the heterogeneity we now compare different issuer profiles on the basis of the following specific quantities of interest:

5.2.1 Overall mobility vs. AAA stability

The Jafry-Schuermann mobility metric proposed in Jafry and Schuermann (2004) is a measure of overall mobility for a rated issuer. It is the average of the singular values of the mobility matrix for that issuer profile. Here, the mobility matrix is obtained by subtracting an identity matrix from the one year transition probability matrix for that issuer profile. For Industrial issuers in the US, our standard profile, this metric is 0.1552092. An analysis of the Jafry-Schuermann mobility metric illustrates that compared to the standard profile, Utility sector issuers are generally less mobile and Banking sector issuers are generally more mobile. We see a pattern consistent with this result when examining the AAA stability i.e. the chance that AAA issuers, which have negligible credit risk, will remain in the AAA rating category after one year. In general these stay probabilities are smallest for Banking issuers and largest for Utility issuers, with those for Industrial issuers lying somewhere in between. Both these observations may have to do with the fact that there is usually much less uncertainty about the revenue streams of Utility sector issuers (especially if they are regulated monopolies). Banking sector issuers are generally highly leveraged and their future revenue streams usually have higher variance.

5.2.2 C→D default probability

Ideally we would like to compare unconditional one year default probabilities across issuer profiles. However, the proportions of issuers across rating categories vary across profiles, and the overall default probability becomes a difficult object of comparison. In general the largest default probability is from the C rating category. Hence we compare and contrast default behavior using C→D default probability. Compared to other issuer profiles the standard profile of US Industrial issuers shows a generally higher default probability and may lead to an overestimation of default probabilities if issuer heterogeneity is ignored. Within each sector, the ordering observed for the C→D default probability is UK > Canada > US > EU > Japan.

It is possible that this clear ordering is a reflection of the differences in corporate bankruptcy environment across different countries. Davydenko and Franks (2006) claim that “despite significant adjustments in lending practices, bankruptcy codes still sharply affect default outcomes”. Furthermore, Mayer (1998) notes that “there are important differences in corporate systems across countries”. In fact, the ordering of countries we observe is also consistent with the ordering of countries by their accounting standards as obtained from Table 5 of Porta, de Silanes, Shleifer, and Vishny (1998). One possible explanation for this ordering is therefore that the stance a rating agency may take in granting a C rating to a debt issuer is to be overly conservative in a country with lenient accounting standards (as was the case with Japan in the past), thus artificially inflating the C rated issuer base. Thus the observed proportion of C rated defaulters would be lower. In contrast, it could be much more lenient in a

country with conservative accounting standards (e.g. UK), resulting in the observed proportion of C rated defaulters to be much higher.

5.2.3 BAA upgrade probability

Similarly, one can examine the probability that a BAA rated issuer is upgraded to either AAA, AA or A rating category within the next year. Analyzing this probability shows that the Banking sector issuers are 10-15% more likely to be upgraded than issuers from other sectors. Possibly, this may be due to the relative opacity of financial firms that prompts rating agencies to take a more conservative stance and assign a lower rating initially, followed by an upgrade in due time. Furthermore, UK issuers systematically have a higher upgrade probability than US issuers. The latter observation may simply be a consequence of the higher mobility of UK issuers, since they also demonstrate a higher C→D default probability than US issuers.

6 Implications for Risk Capital

Risk capital is the amount of capital kept aside to cover unexpected economic losses during extreme events. We offer a small illustration of how issuer heterogeneity affects risk capital. We construct a hypothetical “typical” credit portfolio, then compute the loss distribution on this portfolio with and without incorporating issuer heterogeneity. The two loss distributions give rise to two different estimates for risk capital, which we choose to quantify by the difference between Value at Risk (VAR) and Expected Loss (EL) for the portfolio at hand. It turns out that for this particular portfolio the risk

capital is higher if heterogeneity in default rates is ignored.

The “typical” hypothetical portfolio construction was guided by the following considerations. First, the number of obligors should be approximately 100. Second, the industry sector concentration of exposure amounts should roughly mirror the sector-wise distribution of loan amounts tabulated in Heitfield, Burton, and Chomsisengphet (2006).³ Third, the distribution of credit quality should roughly be 15% good, 60% medium and 25% bad. The actual portfolio constructed deviated from these considerations slightly but more or less respected all the preset criteria (e.g. it had 105 obligors instead of 100). We assumed the recovery rate to be constant at 40%. The total nominal amount of exposure does not matter as we are interested in risk capital as a percentage of that amount.

The model and method we proposed so far was to estimate the default risk is applicable at obligor level. In a portfolio setting, the dependence structure of defaults becomes crucial in determining the loss distribution of the overall portfolio. We used CreditRisk+ to model this dependence structure. We considered two scenarios. First the default rate inputs were chosen to differ across obligors depending on which industrial sector they lie in, thus explicitly incorporating heterogeneity. Second the default rate inputs were input as if all obligors belonged to the standard profile of Industrial issuers, thus assuming homogeneity.

³Admittedly, the loan default rates may differ significantly from those for corporate bond issuers, but a similar sector-wise decomposition for a ‘typical’ bank’s corporate bond portfolio was not available from publicly available sources. Given this shortcoming, we report results for other portfolio constructions as well. We confirm that unless the bank’s portfolio comprises entirely of issuers in the standard issuer profile, heterogeneity matters even if the portfolio does not mirror this ‘typical’ portfolio.

We found that ignoring sector heterogeneity in default rates increased the risk capital from 5.3% to 6.1% which is an increase of about 15% in proportional terms. Choosing median loss instead of expected loss to define risk capital, or choosing sector-specific recovery rates instead of a universal constant 0.4, did not change this result in any significant way. Upon choosing a well diversified benchmark portfolio with equal weights across sectors, the impact of heterogeneity on risk capital increased further (to 20% in proportional terms). The weight of US Industrial issuers in the equal-weights portfolio was lower than in the ‘typical’ portfolio. Hence a larger component of the portfolio deviated from the standard profile. Upon choosing a portfolio with high concentration, the impact depended on the sector it was concentrated in. For concentration in the US Utility sector, which has much lower default rates than the standard profile, the impact of heterogeneity was to increase the risk capital by 24% in proportional terms. Overall, it seems that default rate heterogeneity impacts risk capital and the magnitude of impact is not negligible. This result may have policy implications and requires further investigation.

7 Conclusion

Using a continuous time model, Bayesian estimation techniques and a sample of roughly 23000 rating transitions from the Moodys corporate bond default database we identified strong differences in rating migration behavior between issuers of different industry sectors and countries. Quantifying these differences via rating mobility/stability and also via default/upgrade probabilities yielded several systematic deviations (from

estimates for the standard issuer profile viz. US Industrial issuer). This provides strong support and a tool to condition generator estimates on issuer profiles. In a portfolio context, such a conditioning gave a clearer picture of Value at Risk and risk capital. In particular, it highlighted the possibility that ignoring heterogeneity may increase risk capital by a large percentage. This may be an issue worth examining further, given the upcoming Basel II implementation.

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Appendix - Truncation technicalities

If the issuer was rated before the start of the observation period, then the initial transition $T_{i,0}$ precedes a , the start of the observation period, and has rating $s_{i,0}$. The rating continues in state $s_{i,0}$ from a to $T_{i,1}$. If the issuer is first rated after a , then $T_{i,0}$ and $s_{i,0}$ are not defined. Similarly, some of the durations are right truncated. If the issuer does not default, then the $n_i + 1$ transition occurs at time $T_{i,n_i+1} > b$ where b is the end of the observation period. If the issuer defaults in the observation period,

then the process ends at the last transition T_{i,n_i} because default is an absorbing state, and T_{i,n_i+1} is undefined. The transition times $T_{i,0}$ and T_{i,n_i+1} , when they are defined, are not observed.

If the issuer is rated before a , the beginning of the observation period, then the initial duration is $D_{i,0} = T_{i,1} - T_{i,0}$. However, we only observe the left truncated duration $D_{i,0}^* = T_{i,1} - a$. If the issuer is first rated after a , then $D_{i,0}$ is undefined. If the issuer does not default in the observation period, then D_{i,n_i} is right truncated, and we only observe $D_{i,n_i}^* = b - T_{i,n_i}$. If issuer i defaults before b , then D_{i,n_i} is not defined because default is an absorbing state. If issuer i was rated before a , the start of the observational period, we observe the left truncated duration $D_{i,0}^* = T_{i,1} - a$. Using the memoryless property of the exponential distribution, $D_{i,0}^*$ also has an exponential distribution with rate $\exp(-y_{i,0})$. If the issuer is not in default over the observation period, then D_{i,n_i}^* is right censored, and its contribution to the likelihood function is:

$$P(D_{i,n_i}^* > b - T_{i,n_i}) = \exp[-\exp(-y_{i,n_i})(b - T_{i,n_i})].$$

Table 1: Daily Generator (Q) & Yearly Transition Probability Matrix (TPM) Estimates for the Standard Profile i.e. US Industrial Issuers

Q	AAA	AA	A	BAA	BA	B	C	WR	D
AAA	-2.977E-04	1.967E-04	1.141E-05	4.226E-15	1.358E-13	2.724E-14	3.205E-16	8.962E-05	9.690E-10
AA	2.846E-05	-3.793E-04	2.273E-04	3.307E-06	2.350E-06	1.881E-06	4.269E-08	1.157E-04	1.687E-07
A	1.022E-06	3.748E-05	-3.136E-04	1.598E-04	1.048E-05	2.515E-06	1.274E-07	1.022E-04	4.843E-08
BAA	2.782E-07	3.793E-06	8.438E-05	-3.947E-04	1.564E-04	1.414E-05	4.553E-07	1.345E-04	7.892E-07
BA	7.691E-08	1.929E-06	9.280E-06	1.093E-04	-6.692E-04	2.932E-04	1.165E-05	2.398E-04	3.953E-06
B	1.987E-07	1.937E-06	4.980E-06	1.115E-05	1.330E-04	-7.154E-04	3.030E-04	2.318E-04	2.925E-05
C	1.409E-10	1.295E-09	3.740E-07	7.151E-06	1.457E-05	1.445E-04	-1.043E-03	2.618E-04	6.150E-04
WR	1.571E-07	4.988E-07	2.064E-06	2.461E-06	4.851E-06	5.386E-06	1.550E-06	-1.697E-05	4.080E-10
D	0	0	0	0	0	0	0	0	0

Table 2: Estimation Results : Transition Probability Matrices (over one year horizon) for US Issuers in various industry sectors. Table entries are 100 times the probability values.

Banking Sector	AAA	AA	A	BAA	BA	B	C	WR	D
AAA	86.09	10.43	0.32	0.01	0.00	0.00	0.00	3.15	0.00
AA	0.31	90.36	5.30	0.09	0.01	0.03	0.00	3.90	0.00
A	0.05	5.06	88.69	2.55	0.15	0.03	0.00	3.46	0.00
BAA	0.13	1.59	14.50	72.95	5.23	0.43	0.04	5.10	0.03
BA	0.02	0.71	5.25	14.15	62.83	10.30	1.20	5.48	0.06
B	0.07	0.10	0.82	4.11	13.60	57.96	13.83	8.62	0.89
C	0.00	0.00	0.03	0.20	0.27	0.72	85.50	2.65	10.62
WR	0.04	0.10	0.14	0.12	0.03	0.01	0.00	99.56	0.00
D	0	0	0	0	0	0	0	0	100
Industrial Sector	AAA	AA	A	BAA	BA	B	C	WR	D
AAA	89.74	6.35	0.64	0.02	0.01	0.01	0.00	3.24	0.00
AA	0.92	87.16	7.32	0.32	0.10	0.07	0.01	4.10	0.01
A	0.04	1.21	89.32	5.14	0.47	0.11	0.01	3.70	0.00
BAA	0.01	0.14	2.72	86.76	4.73	0.68	0.05	4.87	0.04
BA	0.00	0.07	0.35	3.32	78.63	8.36	0.76	8.26	0.25
B	0.01	0.06	0.17	0.43	3.82	77.45	8.05	8.06	1.95
C	0.00	0.00	0.02	0.22	0.50	3.87	68.54	8.14	18.71
WR	0.01	0.02	0.07	0.09	0.16	0.18	0.06	99.40	0.01
D	0	0	0	0	0	0	0	0	100
Utility Sector	AAA	AA	A	BAA	BA	B	C	WR	D
AAA	94.62	3.49	1.67	0.04	0.00	0.00	0.00	0.17	0.00
AA	0.32	91.12	6.99	0.38	0.01	0.01	0.00	1.16	0.00
A	0.02	1.67	92.57	4.07	0.15	0.01	0.00	1.50	0.00
BAA	0.02	0.07	3.41	91.24	2.96	0.22	0.10	1.96	0.02
BA	0.02	0.07	0.35	9.37	83.47	3.70	0.90	1.98	0.13
B	0.01	0.02	0.07	2.59	14.92	67.44	9.00	4.12	1.83
C	0.04	0.00	0.11	2.01	3.48	11.38	63.29	7.89	11.80
WR	0.00	0.02	0.09	0.16	0.03	0.01	0.01	99.68	0.00
D	0	0	0	0	0	0	0	0	100