CDO under Basel II: do ratings provide the right size for risk?

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Overview

- Scope of the paper
  - Purposes and hypothesis
- Regulatory treatment
  - Rating based Approach (RBA)
- Methodology
  - The loss distribution
  - Risk measures for CDO
- Comparison and results

Scope of the paper

Tranche	Ratings	Spread (bp)
$0-3$	NA	500+27%
$3-6$	A-	176
$6-9$	**AAA**	**97**
$9-12$	AAA	74
$12-22$	**AAA**	**46**

..the spread of equally rated tranches is quite different..

Is it correct to assign the same capital charge?

We focus on a bank investing (i.e. selling protection) in iTraxx tranches. Our aim is:

1. determine the capital requirements under different approaches introduced by Basel II for structured credit
2. provide a consistent measure for the risk of tranched products
3. **determine the reliability of rating as risk indicators for structured products**

Regulatory treatment

- for structured credit products, the Basel Committee appointed rating agencies as supervisors' "delegated monitors", linking capital charges to credit ratings
- different approaches for standard and advanced banks (IRB banks) and for banking and trading book
- **IRB Banks**:
  - should a rating exist, the tranche capital charge (CC) comes from a coefficient which reflects the agency's view on the transaction creditworthiness (*Rating Based Approach*, RBA);
  - for unrated products the CC is instead the output of a formula set by regulators (*Supervisory Formula Approach*, SFA) which requires banks to provide their internal estimates of probability of default (PD) and recovery rate (1-LGD) for the transaction underlying assets.
    - based upon the *Uncertainty in loss Prioritization* model (ULP) developed by Gordy and Jones (2003) the SFA allocates the capital requirements of a given portfolio (K_{irb}) along the different tranches of a structured credit

RBA risk weights

Long Term Rating	Tranche type		
	Senior	*Base case*	*Backed by non granular pools*
Aaa/AAA	7%	12%	20%
Aa/AA	8%	15%	25%
A1/A+	10%	18%	35%
A2/A	12%	20%	
A3/A	20%	35%	
Baa1/BBB+	35%	50%	
Baa2/BBB	60%	50%	
Baa3/BBB-	100%		
Ba1/BB+	250%		
Ba2/BB	425%		
Ba3/BB-	650%		
Ba3/BB-Unrated	*Deduction*		

- Peretyatkin and Perraudin (2004) analyzed the robustness of the RBA risk weights with respect to different maturities, granularities, and default correlations. They found prudential requirements broadly consistent with the model based CC

Methodology

- let $Value(t_0)$ be the value of a single tranche CDO at the inception date

$$Value(t_0) = MtM(t_0) = fee(t_0) - contingent(t_0)$$

- the *expected loss* is the key input to evaluate both the fee and the contingent leg
- we adopt the market standard *One Factor Gaussian Copula Model* (OFGC) based upon Merton's model
- the portfolio loss distribution is computed using the analytical method described by Andersen, Sidenius and Basu (2003)

Loss Distribution

- let $A_i = wX + \sqrt{1-w^2}\epsilon_i$ be the evolution of firm asset under the OFGC
- starting from the PD for the i-*th* company we derive the conditional PD as

$$q_i(t|X=x) = \Phi\left(\frac{\Phi^{-1}(PD_i(t)) - \sqrt{\rho}x}{\sqrt{1-\rho}}\right)$$

- we compute the entire conditional default distribution $p(l,t|X)$ $(l = 0, ..., N)$ using the iterative algorithm proposed by Andersen, Basu and Sidenius

$$\begin{aligned}
p^{k+1}(0,t|X) &= p^k(0,t|X)(1-q_{k+1}(t|X)) \\
p^{k+1}(l,t|X) &= p^k(l,t|X)(1-q_{k+1}(t|X)) + p^k(l-1,t|X)q_{k+1}(t|X) \\
p^{k+1}(k+1,t|X) &= p^k(l,t|X)q_{k+1}(t|X)
\end{aligned}$$

- ***Heterogenous approach***: we consider the actual number of obligor in the pool as well as the individual PDs. We maintain the hypothesis of constant recovery rate R (R=40%)

Loss Distribution

provided the index components share the same notional amount and the same recovery rate, the loss distribution will assume discrete values. We can easily derived the tranche expected loss

$$E_t[L] = \sum_{l=0}^{N} p(l,t)\{min(lA(1-R), H_b) - min(lA(1-R), L_b)\}$$

- l refers to the number of defaults observed by time t
- H_b and L_b are respectively the tranche detachment and attachment points
- N is the number of exposures composing the underlying portfolio ($N = 125$ for the iTraxx index)

Risk Measure - *UL*

- an ideal candidate to compute the economic capital is the *Credit* VaR derived from the gaussian copula model

$$UL = E_1^{RW}[L|X = x_{99.9}] =$$

$$\sum_{l=0}^{N} p^{RW}(l,t|X = x_{99.9})\{min(lA(1-R), H_b) - min(lA(1-R), L_b)\}$$

- *Pros*
  - consistent with the Basel II treatment for credit exposures
  - easy to compute: just taking the expected loss over 1-year time horizon conditioned at the $99.9 - th$ percentile of the systematic factor
- *Cons*
  - UL does not consider the Mark to Market effect which arises in the case of instruments with maturity > 1 year
  - it does not consider the spreads cashed-in by the protection seller

Risk Measure - UL_{mod}

- we tried to develop a more comprehensive approach, considering both losses arising in $t_1 - t_0$ and that which occurs when the impaired securities are marking to market in t_1

$$UL_{mod} = E_1^{RW}[CashFlow(t_0, t_1)|X = x_{99.9}] + |MtM^{RN}(t_1, t_5)|$$

- the first term largely resembles the UL from which it diverges in considering the money cashed in by the protection seller
- the second one accounts for the mismatch between the time horizon of the VaR measure and the tranche maturity. Its evaluation is based on the pricing of a 4-year *forward starting CDO* written on the original pool starting at t_1
  - we applied the pricing methodology developed for *specific forward starting CDOs*
  - Hull & White (2007) showed that such a *specific forward CDO* can be fairly priced by the One Factor Gaussian Copula Model

Risk Measure - UL_{mod}

let's consider MtM

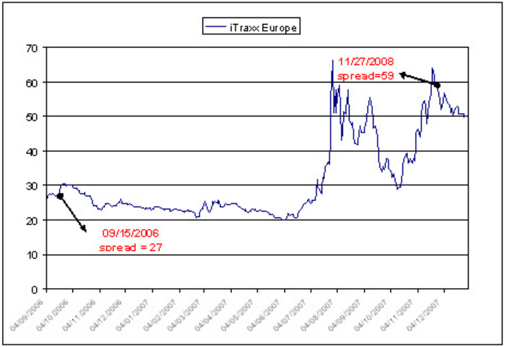
- in determining the MtM at t_1 we are interested only in cases where losses accrued during the first year, L_1, have not exhausted the tranche
- we applied the methodologies described by Hull & White after reducing attachment and detachment points by L_1. In doing so we got the attachment and detachment points: $L_b^* = L_b - L_1$ and $H_b^* = H_b - L_1$.
- Moreover in computing the probability distribution of default $p^{RN}(l,t)$ from t_1 to t_5 the portfolio composition should be updated by removing the defaulted companies. Assuming $L_1 = l^* A(1-R)$, we estimated the number of defaults to be considered for this purpose as

$$l^* = \frac{L_1}{A(1-R)}$$

- if $l^* = 6$ the remaining portfolio will count 119 obligor. As we do not know which particular companies have defaulted, we adopt the Large Homogeneous Pool approach in computing $p^{RN}(l,t)$.

Set up comparison

- focus on standard tranches quoted on the iTraxx index, considering the 5-y maturity
- analysis performed over two different period



Set up comparison

September 15, 2006 - *Low volatility*		
5Y	Index spread 27	
Tranches	Mid Spread (bp)	Base Corr
0 − 3	500+15.65%	12.7%
3 − 6	49.5	22.5%
6 − 9	14.5	29.7%
9 − 12	6.5	35.4%
12 − 22	3	51.3%

November 27, 2007 - *High volatility*		
5Y	Index spread 58	
Tranches	Mid Spread (bp)	Base Corr
0 − 3	500+27.65%	32.9%
3 − 6	176	48.8%
6 − 9	97	58%
9 − 12	74	64%
12 − 22	46	78%

Table: Source: Bloomberg

- actual PDs come from the issuer rating assigned by Fitch to the 125 iTraxx companies
- risk neutral PDs backed from CDS market
- we considered three different values for the correlation $\rho = 15\%, 25\%, 35\%$
- RBA charges are based on model implied ratings obtained by Fitch proprietary model for synthetic CDO, Vector 3.2. We also consider the new proposal, Vector Beta
- for the SFA, the K_{irb} parameter was estimated using actual PD and a fixed LGD of 60%

Results

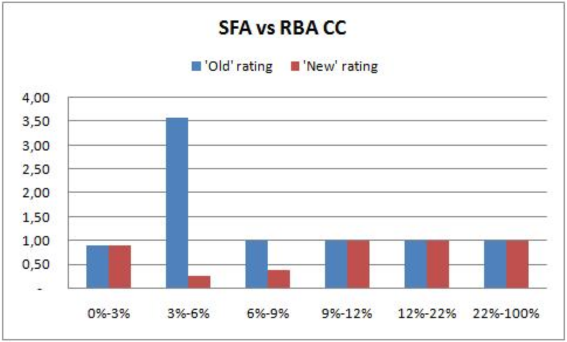
- new ratings cause a downgrade for junior-mezzanine and mezzanine tranche

Tranche	Ratings (*Old*)	RBA (*Old*)	Ratings (*New*)	RBA (*New*)	SFA
0 – 3	NA	100.00	NA	100	92.00
3 – 6	A-	2.80	BB	36.00	10.00
6 – 9	AAA	0.56	AA-	1.44	0.56
9 – 12	AAA	0.56	AAA	0.56	0.56
12 – 22	AAA	0.56	AAA	0.56	0.56
22 – 100	NA	0.56	NA	0.56	0.56

- tranches rating are the same for both periods we considered. The rolling mechanism of the index, replacing riskier companies with safer ones, tends to keep the portfolio average quality constant

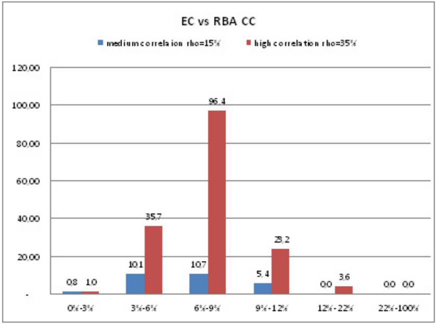
Results - SFA *vs* RBA

- when *old ratings* are considered RBA and SFA charges appear to be closely aligned only at boundaries of the capital structure. The alignment relaxes moving to the center of the capital structure as SFA CC becomes significantly higher than the RBA ones
- the opposite happens with RBA CC stemming from *new Ratings*, providing clear evidence of the additional degree of prudence adopted when developing the latest Vector release



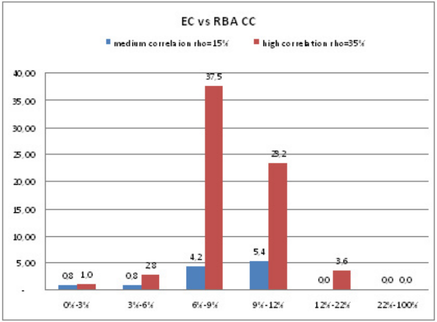
Results - CC *vs* EC, Old ratings

- **RBA CC, expected to be the preferred criteria for CDOs, do not fully cover the economic risk**
- when considering old ratings economic capital (UL_{mod}) fairly exceed RBA charges for all tranches but the equity
- the gap is sizable especially for the mezzanine investments: given a correlation of 15% the regulatory capital results being 10 times lower than the EC (for the $6-9\%$ tranche). If we consider the highest correlation, 35%, the ratio jumps to 96.



Results - CC *vs* EC, New ratings

- regulatory and economic capital are more aligned using *new ratings*. In this case the ratio between EC and RBA CC plummets from 10 to about 4 for the $6\% - 9\%$ ($\rho = 15\%$), but widens again as we let the correlation to increase.



Results - Correlation

- the magnitude of the gap does not change moving from the low volatility period to the high volatility one
- ***pay attention to the correlation***: in normal times a value between 15% and 25% could be viewed as a fair choice, whilst a higher value (i.e. 35% or more) is more appropriate in bad conditions.

Tranche	$LowVol$			$HighVol$		
	ρ					
	15%	25%	35%	15%	25%	35%
$0-3$	76	100	100	72	100	100
$3-6$	28	63	100	29	60	100
$6-9$	6	17	54	9	20	52
$9-12$	3	4	13	4	7	15
$12-22$	0.7	1	2	0.7	1.5	3
$22-100$	0	0	0	0	0	0

Conclusions

- ratings are discrete in nature and address just one dimension of structured products risk, the probability of default
- they are not the right tool for capital allocation, that should be based on tailor made metrics linked to extreme values of the loss distribution.
- our results are influenced by the choice of adopting the Basel II capital horizon for banking exposures of one year.
- it would be useful to enlarge the spectrum of the analysis by considering less stylized corporate CDOs or other structured credit products, like CDOs of ABS. In the event the same findings emerge it should be questioned whether the Basel II mapping from ratings to capital is the right choice or it is encouraging new forms of arbitrage.