CDO under Basel II: do ratings provide the *right size* for risk?*

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The huge development experienced by the market for structured credit products has occurred under a banks' capital adequacy regime inadequate to deal with the complexity of these instruments. In this paper we try to evaluate the reliability of Basel II treatment for securitization exposures, comparing the forthcoming charges for structured products with an estimate of their risk. The results obtained cast serious doubts about the actual degree of conservatism of the choice to map from ratings to prudential charges.

I. INTRODUCTION

The recent turmoil in credit markets has raised diffused concerns regarding the reliability of credit ratings as effective tools for assessing the risks of structured credit products. For regulators the alarm bells rang even louder since Basel II rules for securitisation exposures link to public ratings the amount of capital that banks have to set aside when investing in ABS, CDOs and the like.

In this paper we investigate the robustness of the new rules comparing capital charges for structured credit products held into the banking book with an estimate of the economic capital absorbed by such investments. The analysis focuses on iTraxx tranches, a stylized version of corporate CDO whose active market allowed us to observe the impact of recent turmoil on both economic and regulatory measures.

We combine together information conveyed by market data with those obtained by Fitch's ratings, used to assess the risk of CDO tranches and the creditworthiness of firms comprised into the iTraxx.

In order to assign a rating to the tranches, the Fitch's methodology, Vector 3.2, has been used. The availability of a "proposal review" of the same approach allowed us to compare the output of two models so to get some preliminary insights into the prudential "side-effects" of the initiatives that all main rating agencies are now undertaking to strengthen CDOs evaluations.

After providing an overview of the new prudential rules for securitisation products, we describe the approach used to estimate CDOs risks; the results of the comparison with prudential charges are then shown, while the final section reports our conclusions.

II. REGULATORY TREATMENT

The ultimate goal of Basel II framework is to promote the adequate capitalization of banks and to encourage improvements in risk management, thereby strengthening the stability of the financial system.

The new regulation provides a set of rules aiming to achieve a better alignment between regulatory charges for credit exposures and their economic capital absorbtion (EC), i.e. the amount of capital that could be lost in bad market conditions.

By deviating from a general openness to prudential use of internally developed risk measures, in the case of structured credit products, the Basel Committee appointed rating agencies as supervisors' "delegated monitors", linking capital charges to credit ratings [1]. This holds true even for the most sophisticated banks, those qualified to use internal assessments of credit exposures as the basis for capital requirements.

For these firms, collectively known as IRB (Internal Rating Banks), two alternative approaches have been set:

- should a rating exist, the tranche capital charge (CC) comes from a coefficient which reflects the agency's view on the transaction creditworthiness (*Rating Based Approach*, RBA)[1];
- for unrated products the CC is instead the output of a formula set by regulators (*Supervisory Formula Approach*, SFA) which requires banks to provide their internal estimates of probability of default (PD) and recovery rate (1-LGD) for the transaction underlying assets.

Both RBA and SFA charges are intended to cover expected and unexpected losses, since in the case of securitisation products the large part of expected losses tends to gather in junior tranches.

Every time a credit rating is available or could be inferred, the compulsory approach RBA provides a simple solution for mapping from rating letters to risk weights (table I): coefficients are all the same for every securitisation product, the only differences being pool granularity and tranche seniority. More precisely if a tranche

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*The view expressed in the study are those of the authors and do not involve the responsibility of the institution to which they belong.

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[1] Basel II regulatory charges result from a twofold weighting process, where exposures are first multiplied by a specific "risk weight" and then by the 8%, common to all positions; to make comparisons easier we transformed risk weights in capital charges, defined as the percentage of capital banks have to set aside for each unit invested.

is the most senior in its structure and enjoys an investment grade rating, it attracts a lower risk weight than the base case, as long its underlying pool is perceived to be sufficiently granular[2].

Rating	Tranche type		
	senior tranches	*base case*	*non granular pools*
AAA	0.56%	0.96%	1.60%
AA	0.64%	1.20%	2.00%
A+	0.80%	1.44%	2.80%
A	0.96%	1.60%	2.80%
A-	1.60%	2.80%	2.80%
BBB+	2.80%	4.00%	4.00%
BBB	4.80%	6.00%	6.00%
BBB-	8.00%	8.00%	8.00%
BB+	20.0%	20.0%	20.0%
BB	36.0%	36.0%	36.0%
BB-	52.0%	52.0%	52.0%
Below BB-	100.00%	100.00%	100.00%

TABLE I: RBA Capital Charges (%)

The robustness of RBA weights has been challenged by Peretyatkin and Perraudin[12] that used a simulation approach to calculate marginal Value at Risk values for ideal tranches held into a well diversified portfolio. They analyzed the impact of different maturities, granularities, default correlations and found prudential requirements broadly consistent with the model based CC.

Thought as an emergency exit for unrated tranches held by banks that originated the underlying assets, the SFA tends primarily to disincentive regulatory arbitrages schemes that became so popular under Basel I. Its goal is hence to deliver "capital neutrality" when allocating to different tranches the regulatory charge attracted by the underlying credit exposures (K_{IRB}, i.e. the capital that an IRB bank should set aside against the securitised assets if they were held on balance-sheet).

The basic idea for the formula came from Pykthin and Dev [9], [10] that developed a model based on a strict prioritization rule in losses allocation (SLP, *Strict Losses Prioritization*) which requires the knowledge of only two of the tranche's features: thickness and credit enhancement level.

Gordy and Jones [6] improved the viability of SLP for prudential purposes smoothing out its characteristic cliff effect. This was obtained allowing for the uncertainty that from the supervisors' standpoint surrounds the actual allocation of the pool's cash-flows, governed by contractual rules specific to each transaction (ULP, *Uncertainty in Losses Prioritization*).

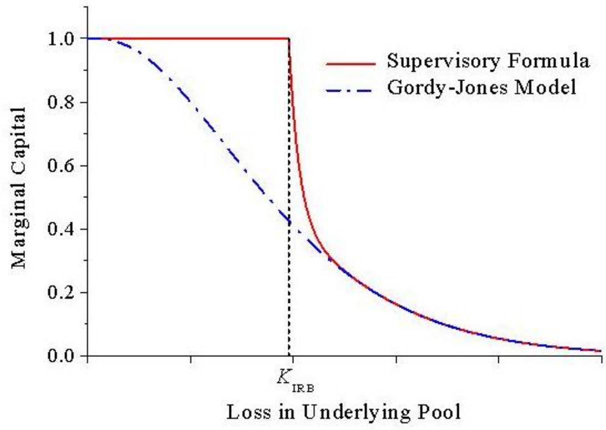

FIG. 1: Supervisory formula

The supervisory formula is based on an approximation of the ULP along with some "supervisory overrides", like a dollar for dollar capital requirements for tranches whose credit enhancement fall short of K_{IRB}, and an imposed floor level for most senior positions (figure 1).

While RBA and SFA charges apply only to banking book items, it could be expected that the forthcoming regulation on the treatment of credit risk arising from trading book items (so-called *Incremental Default Risk*) will ask banks to calibrate their models to banking book charges, so to prevent the insurgence of differences not explained by liquidity considerations.

III. RISK MEASURES

The easiest approach to compute the credit risk of a CDO implies the use of the market standard model for pricing and risk analysis of structured credit products, the *One Factor Gaussian Copula* (OFGC) on which relies also the risk weight function set for credit exposures under Basel II[5].

While the normal copula provides us a general framework for addressing default correlation in the underlying pool, it became popular between practitioners to restrict the dependence structure to the one described by a single common factor.

Specifically, under the OFGC the credit standing of each asset is supposed to be driven by an associated normal variable $A_i = wX + \sqrt{1-w^2}\epsilon_i$ where both X and ϵ have a standard normal distribution.

The correlation between any two names is then equal to the square of the factor loading: $\rho = corr(A_i, A_j) = w^2$.

In the context of the OFGC model with ρ we refers to the *asset correlation*. Given the asset correlation it is possible to calculate the default correlation analytically.

[2] A pool is said to be highly granular if it contains a large number of exposures, none of which contributes a large part of the total risk.

The default correlation results in a strict monotonous increasing function of the asset correlation.

While this approach provides only a rough description of interdependence between firms into the pool, it has the advantages of simplicity and tractability: conditionally to the common factor defaults are independent which makes the computation of the probability distribution quite simple[3].

The random variable A_i is often interpreted in the context of a Merton model[11], as the value of a company's assets. So, given the PD for the i-*th* company we can then recognize the threshold α as:

$$A_i = wX + \sqrt{1-w^2}\epsilon_i < \alpha \equiv \Phi(PD_i(t)) \quad (1)$$

setting a fixed value for X let us to derive the conditional PD:

$$q_i(t|X=x) = \Phi\left(\frac{\Phi^{-1}(PD_i(t)) - \sqrt{\rho}x}{\sqrt{1-\rho}}\right) \quad (2)$$

The simplest way to move from the individual conditional PD to the pool loss distribution is known as the *Large Homogeneous Pool* (LHP). It assumes the portfolio being composed of infinitely small positions (*perfect granularity*), all sharing the same probability of default ($PD_i = PD$), recovery rate ($R_i = R$) and notional amount ($A_i = A$).

A better approximation of reality can be achieved considering the actual composition of the pool (in the case of the iTraxx 125 equally weighted names, i.e. constant A) and the company's PD (PD_i). When adopting this approach in our exercise, so-called *Heterogenous approach*, we slightly deviate from that described by Allen and Beinstein[2] assuming a constant recovery rate R, conventionally fixed at 40%.

It is common practice to compute the entire conditional default distribution $p(l,t|X)$ $(l = 0,...,N)$ using the iterative algorithm proposed by Andersen, Basu and Sidenius [13]; by integrating over the systematic factor we can hence obtain the unconditional distribution, $p(l,t)$.

Under this framework the tranche expected loss, the key input for both pricing and risk evaluation, is equal to

$$E_t[L] = \sum_{l=0}^{N} p(l,t)\{min(lA(1-R),H_b) - min(lA(1-R),L_b)\} \quad (3)$$

where l refers to the number of defaults observed by time t, H_b and L_b are respectively the tranche detachment and attachment points, and N is the number of exposures composing the underlying portfolio ($N = 125$ for the iTraxx index).

Unexpected loss - UL

A first measure of tranches credit risk can be obtained computing the expected loss at one year time horizon (hereon "capital horizon"), conditioned at the desired percentile of the systematic factor (in our exercise we adopt the $99.9 - th$ percentile).

For this purpose we have to solve (3) considering the probability distribution conditioned to $X = x_{99.9}$:

$$E_1^{RW}[L|X = x_{99.9}] = \quad (4)$$

$$\sum_{l=0}^{N} p^{RW}(l,t|X = x_{99.9})\{min(lA(1-R),H_b) - min(lA(1-R),L_b)\}$$

$E_1^{RW}[.]$ can be considered as a measure for the *Unexpected Loss* computed over the chosen time horizon. The suffix RW stands for "Real World" (RW) and indicates the usage of default probabilities derived from historical experience, that is the preferred choice for credit risk analysis.

While considering losses arising from defaults occurring during the capital horizon, this *Credit VaR* fails to weigh the additional risk resulting from the longer maturity of tranches. This is easily understood if we consider two different tenors but otherwise identical tranches, one with a one-year maturity (t_1) and the other with a five-year maturity (t_5): if we compute (4) at t_1 we end up with the same unexpected loss for both securities.

Modified Unexpected loss - UL_{mod}

For traditional exposures, the New Capital Accord imposes to IRB banks to estimate CC considering the mismatch between the exposure's maturity and the capital horizon (so-called *maturity adjustment*).

In our case including the *maturity adjustment* requires to develop a more comprehensive approach, able to address the total loss suffered by a hypothetical investor who sold protection on a tranche at t_0 and closed the deal in t_1. Such an approach should encompass both the risk arising from losses induced by eventual defaults occurred in $t_1 - t_0$ and that which occurs when marking to market the impaired tranche in t_1.

For this purpose we computed the following measure (*Modified Unexpected Loss*):

$$UL_{mod} = E_1^{RW}[CashFlow(t_0,t_1)|X = x_{99.9}] + |MtM^{RN}(t_1,t_5)| \quad (5)$$

The first term largely resembles the unexpected loss as defined in (4), from which it diverges in considering the money cashed in by the protection seller.

The second one accounts for the mismatch between the time horizon of the *VaR* measure and the tranche maturity. Its evaluation is based on the pricing of a 4-year *forward starting CDO* written on the original pool starting at t_1.

Since we are dealing with pricing, in this case risk neutral probabilities (RN) delivered from market data should be adopted instead of real world ones.

Hull & White[15][14] showed that such a *specific forward CDO*[4] can be fairly priced by the One Factor Gaus-

[3] Two articles by Li [7][8]are cited as the first application of the model to pricing CDOs.

[4] Forward starting CDOs are categorized according to the exis-

sian Copula Model.

In determining the MtM at t_1 we are interested only in cases where losses accrued during the first year, L_1, have not exhausted the tranche. We assumed L_1 equal to the *Unexpected loss* at time t_1 computed by (4).

Consequently, we applied the methodologies described by Hull & White after reducing attachment and detachment points by L_1. In doing so we got the attachment and detachment points as[5]: $L_b^* = L_b - L_1$ and $H_b^* = H_b - L_1$.

As one can easily imagine this results into a reduction of the credit enhancement enjoyed by each tranche so that, for example, we can think of the forward start 3%-6% tranche as being very much like to a spot equity one.

Moreover in computing the probability distribution of default $p^{RN}(l,t)$ from t_1 to t_5 the portfolio composition should be updated by removing the defaulted companies. Assuming $L_1 = l^* A(1-R)$, we estimated the number of defaults to be considered for this purpose as

$$l^* = \frac{L_1}{A(1-R)} \tag{6}$$

For example if $l^* = 6$ the remaining portfolio will count 119 obligors. As we do not know which particular companies had defaulted, in computing $p^{RN}(l,t)$ we adopt the Large Homogeneous Pool approach.

Credit Rating, Vector 3.2

Ratings are by far the most common measure of credit risk for debt securities. In case of structured credit products they largely reflect the outputs of analytical models, even if an important role is played by the assessment of the transaction's structural features delivered by the rating committee.

In our exercise we used the well-known Fitch approach for CDOs evaluation, Vector [3], which maps the probability of each tranche being impaired on the historical default frequency of corporate bonds. Losses in the underlying pool are generated by a multi-step Monte Carlo simulation.

Addressing the probability of tranches being impaired over their maturity (so-called "first cent loss"), CDOs ratings do not take into account the magnitude of actual losses that could be suffered by a tranche investor.

Moreover, as rating are discrete in nature they can provide only a raw picture of the magnitude of risk embedded in similar securities, like CDO senior tranches.

tence of the underlying portfolio at the trade date: in *specific forward start CDOs* this portfolio exists and may have suffered defaults before the start date occurs; in *general forward start CDOs* a *de novo* portfolio will come into existence at the start date, so that at the trade date its features can only be gauged.

[5] Obviously for the $0-3\%$ tranche L_b is forced to 0.

IV. COMPARATIVE ANALYSIS

We performed our analysis with respect to the standard tranches quoted on the iTraxx index, considering the five years maturity. The comparison has been performed considering two different time periods (see fig. 2): September 15, 2006, a period which epitomizes the lull before the storm (*low volatility* period), and November 27, 2007 which was instead a moment in the fog of the war originated from the US subprime crisis (*high volatility* period).

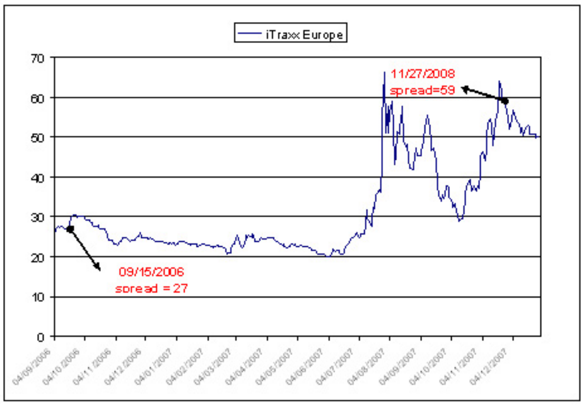

FIG. 2: iTraxx time serie

5Y	Index spread 27	
Tranches	Mid Spread (bp)	Base Corr
$0-3$	500+15.65%	12.7%
$3-6$	49.5	22.5%
$6-9$	14.5	29.7%
$9-12$	6.5	35.4%
$12-22$	3	51.3%

TABLE II: Tranches information at September 15, 2006

5Y	Index spread 58	
Tranches	Mid Spread (bp)	Base Corr
$0-3$	500+27.65%	32.9%
$3-6$	176	48.8%
$6-9$	97	58%
$9-12$	74	64%
$12-22$	46	78%

TABLE III: Tranches information at November 27, 2007

For each tranche the economic capital has been computed adopting the same assumptions used by the Basel

Committee when delivering banking book CCs: one year time horizon and 99.9% confidence level. Results have been then compared with CCs generated by RBA and SFA approaches.

We assumed the economic capital being the capital needed to cover the risk measured under the *Modified Unexpected Loss*, whose calculation required us to gauge default probabilities under both actual and risk neutral measures.

Actual PDs have been derived from *issuer ratings* assigned by Fitch to the 125 companies listed in the series of the iTraxx index that were "on-the-run" at the moment of the analysis. As a proxy for the PD we considered the 5-year cumulative default rate (CRD) relative to long-term debt issuer ratings assigned by Fitch from 1990 to 2006 [4]. From these figures we derived the constant intensity of default as $\lambda = -log(1 - CRD)/5$.

Risk neutral PDs have been backed-out from CDS market. We assumed a Poisson process for default, implying $PD = 1 - exp(-\lambda t)$. For each company the annual default intensity λ has been derived from the quoted 5-year CDS spread; to keep the computation simple we assumed a flat term structure for the default intensity and a constant recovery rate of 40%.

As one could expect actual PDs are, on average, lower that those resulting from from CDS spreads, due to scarcity of defaults in the best rating classes. The average default probability of iTraxx constituents on the five year time horizon reduces by half a percentage point (from 2.45% to 1.91%) moving from the risk neutral world to the real world.

In order to evaluate the sensitivity of proposed risk measure to the correlation coefficient we considered three different values for ρ: 15%, 25% and 35%. We chose to use the same correlation measure for all tranches as the very idea of multiple correlation for the same pool lacks of logical background and is indeed a signal of the limited adequacy of the standard pricing model to explain the quoted tranche prices.

RBA charges are based on model implied ratings obtained using the Vector 3.2 ("Old Ratings"). We also used assessments produced by the latest release of this model that Fitch recently posted to the attention of market participants ("New Ratings").

In implementing the Supervisory Formula we obtained its key parameter, the K_{irb}, using firms' historical PDs and a fixed LGD value of 60% for all names in the pool.

V. RESULTS

Tranches ratings produced by both versions of the Vector model are shown on table IV. Moving from the low volatility period to the high volatility one, all tranches retained their rating letters. This a direct consequence of the rolling mechanism of the index that, replacing riskier companies with safer ones, keeps constant the portfolio average quality.

Tranche	Ratings (*Old*)	RBA (*Old*)	Ratings (*New*)	RBA (*New*)	SFA
$0-3$	NA	100.00	NA	100	92.00
$3-6$	A-	2.80	BB	36.00	10.00
$6-9$	AAA	0.56	AA-	1.44	0.56
$9-12$	AAA	0.56	AAA	0.56	0.56
$12-22$	AAA	0.56	AAA	0.56	0.56
$22-100$	NA	0.56	NA	0.56	0.56

TABLE IV: RBA and SFA Capital Charges

The same holds true for the SFA CCs, given the constancy of the K_{irb} parameter over the two periods.

The comparison between RBA and SFA showed charges being closely aligned for the most senior tranches; likewise, at the lowest end of capital structure the equity tranche faces comparably stringent charges under both approaches (figure 3).

Rather than the result of a sensible calibration of regulatory approaches, this is mainly due to the cap and floor that Basel II applies to the ULP model, which bounds the resulting CC.

The alignment relaxes indeed moving to the center of capital structure, as SFA charges become significantly higher than RBA ones, at least as long as "Old Ratings" are used (3.5 times in the case of the A- tranche).

The opposite happens when the comparison is run on RBA CCs stemming from "New Ratings". This result provides a clear evidence of the additional degree of prudence adopted when developing the latest Vector release.

The new release mainly impacts on the central part of capital structure: causing downgrades of 5 and 3 notches respectively for $3\% - 6\%$ and 6%-9% tranches (table IV) it delivers a sensible increase of CCs (from 2.8 to 36 for the junior mezzanine e from 0.56 to 1.44 for the upper mezzanine).

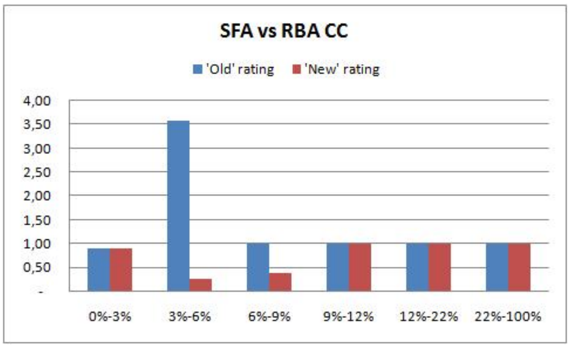

FIG. 3: Ratio between SFA and RBA

Our most relevant finding, however, is that RBA charges, expected to be the preferred criteria for CDOs held into the banking book, do not fully cover the economic risk of tranches.

The magnitude of the gap remains quite similar when moving from the low volatility environment to the high volatility one (table V), since the UL_{mod} is unaffected by an increase in CDS spreads not explained by an actual worsening of credit quality.

This result could be easily understood looking at the way UL_{mod} has been computed. Provided that the portfolio credit worthiness remained unchanged over the two periods, the first term of (5) delivers the same loss amount. With regard to the MtM, the increase of tranches spreads in the high volatility period tends to offset their augmented riskiness.

What drives changes in economic capital across the two periods is instead the asset correlation, whose value should be set so to reflect the conditions of economic environment: in normal times a value between 15% and 25% could be viewed as a fair choice, whilst a higher value (i.e. 35% or more) is more appropriate in bad conditions.

Tranche	$LowVol$			$HighVol$		
	ρ					
	15%	25%	35%	15%	25%	35%
$0-3$	76	100	100	72	100	100
$3-6$	28	63	100	29	60	100
$6-9$	6	17	54	9	20	52
$9-12$	3	4	13	4	7	15
$12-22$	0.7	1	2	0.7	1.5	3
$22-100$	0	0	0	0	0	0

TABLE V: UL_{mod} - CC during the two period

In this respect, it is worthwhile noticing that correlation plays a key role in the computation of our risk measure and explains much of the differences we found between CC and EC. From a regulator's perspective this issue is quite relevant: asset correlation tends to increase during economic downturns, exactly the situation regulatory capital has to cope with.

Figures 4 and 5 show the comparison of CC and EC for different tranches with respect to the low volatility period. They show clearly that our estimates of economic capital absorption (UL_{mod}) fairly exceed RBA charges for all tranches but the equity[6].

The gap is sizable especially for the mezzanine investments: given a correlation of 15% the regulatory capital results being 10 times lower than the EC (for the $6-9\%$ tranche). If we consider the highest correlation, 35%, the ratio jumps to 96.

Regulatory and economic capital are more aligned using "New Ratings". In this case the ratio between EC and RBA CC plummets from 10 to about 4 for the $6\%-9\%$ ($\rho = 15\%$), but widens again as we let the correlation to increase.

[6] Being unrated, the equity tranche is entitled of a dollar by dollar (100%) capital charge.

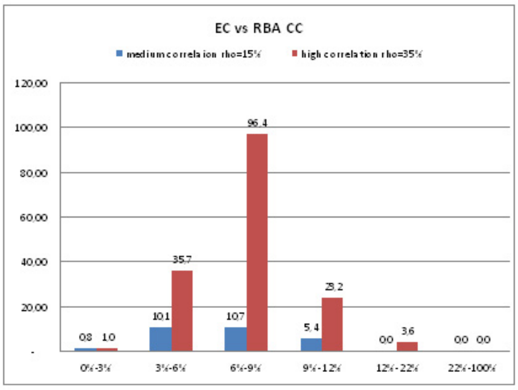

FIG. 4: EC vs RBA ('Old' rating)

Finally, the gap between economic and regulatory capital remains when considering the supervisory formula. By comparing the last column of table IV with the CC reported on tale V, we get the same results presented for the RBA approach.

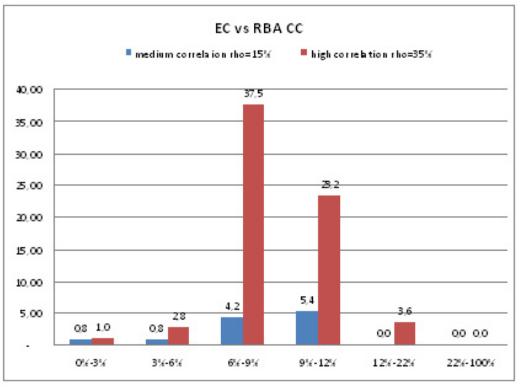

FIG. 5: EC vs RBA ('Old' rating)

Of course our results are influenced by the choice of adopting the Basel II capital horizon for banking exposures of one year. We are aware that this horizon is clearly too extended for iTraxx tranches, that need a much shorter period to be closed or hedged. However this horizon become acceptable as we think to index tranches as a proxy of CDOs of corporate exposures held in the banking book; the scarce liquidity of these products, that the recent upheavals in the market has just confirmed, can well justify the regulators' choice implemented in our exercise.

VI. CONCLUSION

Like polo shirts, that come only in small, medium and large sizes, ratings are discrete in nature and address just one dimension of structured products risk, the probability of default.

For this reason they are not the right tool for capital allocation, that should be based on tailor made metrics linked to extreme values of the loss distribution.

We think that it would be useful to explore the robustness of rating based prudential charges with respect to less stylized corporate CDOs or other structured credit products, like CDOs of ABS.

In the event the same findings emerge it should be questioned whether the Basel II mapping from ratings to capital is the right choice or it is encouraging new forms of arbitrage.

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