Sector Concentration in Loan Portfolios and Economic Capital

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Abstract

The purpose of this paper is to measure the potential impact of business-sector concentration on the economic capital for loan portfolios and to explore a tractable model for its measurement.

In the empirical part we evaluate the increase in economic capital in a multi-factor asset value model for portfolios with increasing sector concentration. The sector composition reflects information from the German credit register data to ensure that they are representative for real banks.

Finding that business sector concentration can substantially increase economic capital we explore in the theoretical part of the paper if this risk can be measured by a more tractable model that avoids Monte Carlo simulations. We analyse a simplified version of the analytic value-at-risk approximation developed by Pykhtin (2004), which only requires risk parameters on a sector level. Sensitivity analyses with various input parmeters show a good perfomance of the model in approximating economic capital for portfolios which are homogeneous on a sector level in terms of PD and exposure size. Furthermore, we explore the robustness of our results for portfolios which are heterogeneous in terms of these two characteristics. We find that low granularity decreases while heterogenity in individual PDs improves the approximation performance, in particular if creditworthiness increases with exposure size. Both effects partly balance each other so that the analytic approximations overall perform reasonably well also for heterogeneous portfolios.

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1. Introduction

Although the missing recognition of diversification within banks' credit portfolios was a key criticism of the 1988 Basel Accord, the minimum regulatory capital requirements (Pillar I) of the revised Basel Framework from June 2004 are based on a single-factor model[3] which still does not account for differences in diversification. However, recognising that banks' portfolios can exhibit credit risk concentrations, Basel II stipulates that this risk should be addressed in the supervisory review process (Pillar II) which creates a need for an appropriate methodology to measure this risk.

Concentration risk in banks' credit portfolios arises either from an excessive exposure to certain names (which is often referred to as *name concentration* or *granularity*) or from an excessive exposure to a single sector or to several highly correlated sectors (i.e. *sector concentration*). Financial regulation and previous research have focused in the past mainly on the first aspect of concentration risk.[4] Therefore, we focus in this paper on sector concentration risk, although we also carry out robustness checks with portfolios of low granularity. Sectors are defined in the following as business sectors. Although geographical regions can also be modelled as sectors, this case is not considered in this paper.

The critical role which credit risk concentration has played in past bank failures has been documented in the literature.[5] Therefore, the importance of prudently managing sectoral concentration risk in banks' credit portfolios is generally well recognised. However, existing literature does not provide much guidance on how to measure sectoral concentration risk or what levels of concentration might merit concern. Consequently, whether or how particular levels of concentration should be translated into an additional capital buffer remains an open question.

This paper contributes to the literature in the following ways. First, we measure economic capital in a CreditMetrics-type multi-factor model and evaluate how important the increase in economic capital is in a sequence of portfolios with increasing sector concentration. The analysis is based on portfolios which were constructed from German central credit register data. They reflect the average business-sector distribution of the banking system as well as higher sector concentrations observed in individual banks. Information on business-sector concentrations of banks is not publicly available and central credit registers represent unique

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[3] See Gordy, M. (2003).

[4] See EU Directive 93/6/EEC, Joint Forum (1993) and Gordy (2003).

[5] See, for example, BCBS (2004a).

sources of data on sector concentrations in existing banks. Our emphasis on empirically observable sector concentrations is therefore an important contribution.

Second, we evaluate the accuracy of a multi-factor adjustment proposed in Pykhtin (2004), which offers a tractable, closed-form solution for value-at-risk (VaR) and economic capital (EC) and, thereby, for the measurement of concentration risk. We have applied a simplified version of the model in order to reduce the computational burden. Such a methodology could be useful for risk managers and supervisors in search of robust, fit-for-purpose tools to measure sector concentration in a bank's loan portfolio.

Our results show that economic capital can substantially increase with sector concentration across portfolios. The increase in economic capital from a credit portfolio which represents the average sector distribution of the German banking system to a portfolio that is concentrated in a single sector can be as high as 50%. This raises the question if sector concentration can be approximated by a tractable and robust method that avoids Monte Carlo simulations. A candidate for such a tool is the model in Pykhtin (2004), which is an important focus of this paper.

The methodological framework of the Pykhtin model builds on earlier work by Gordy (2003) and Wilde (2001) on granularity adjustments in the asymptotic single risk factor (ASRF) model. Whereas the granularity adjustment deals with an unbalanced exposure distribution across names, the Pykhtin model offers a treatment for an unbalanced distribution across (correlated) sectors. The VaR is given in closed form as the sum of the VaR in the single risk factor model and an adjustment term. The model allows banks and supervisors to approximate economic capital for loan portfolios without running computationally intensive Monte Carlo simulations. Furthermore it poses only moderate data requirements since it requires risk parameters only on a sector level.

To our knowledge there is only one recent empirical paper that considers the impact of sector concentration risk on economic capital. Burton et al. (2005) simulated the distribution of portfolio credit losses for a number of real US syndicated loan portfolios. They found that although name concentration can meaningfully increase EC for smaller portfolios (which are defined as portfolios with exposures of less than US-$10 billion), sector concentration risk is the main contributor to EC for portfolios of all sizes.

Two other models that measure concentration risk in a tractable model have been presented by Cespedes et al. (2005) and Düllmann (2006). Cespedes et al (2005) developed an adjustment to the single risk factor model in the form of a scaling factor to the economic capital required by the ASRF model. This "diversification factor" is an approximately linear function of a Hirschmann-Herfindahl index, calculated from the aggregated sector exposures. This model, however, does not allow for different asset correlations across sectors. Contrary to the approach in our paper, it cannot distinguish between a portfolio which is highly concentrated towards a sector with a high correlation with other sectors and another portfolio which is equally highly concentrated, but towards a sector which is only weakly correlated with other sectors. Düllmann (2006) extends Moody's Binomial Expansion Technique by introducing default infection into the hypothetical portfolio on which the real portfolio is mapped in order to retain a simple solution for VaR. Unlike the Pykhtin model, both models developed by Cespedes et al. and Düllmann require the calibration of a parameter using Monte Carlo simulations.

The paper is organised as follows. In Section 2 we present the portfolio model of CreditMetrics and the Pykhtin model. The CreditMetrics model is a multi-factor portfolio model which is very often used by banks to calculate credit loss distributions and economic capital. Default dependencies are captured by means of common factors, which are defined by equity index returns.

The empirical part of our paper comprises Sections 3 and 4. The loan portfolios on which the empirical analyses are based are described in Section 3. To avoid distortions arising from different sector characteristics, we assume all portfolios to have equal exposure to all obligors, a fixed recovery rate and a homogeneous probability of default in every sector. The benchmark for the comparison of portfolio losses is the EC of a loan portfolio, which is defined as the difference between VaR and the expected loss (EL).

In Section 4 we analyse the impact of sector concentration on EC using the default-mode version of the well-established multi-factor CreditMetrics model. From the benchmark portfolio, we gradually increase sector concentration and analyse its impact on economic capital.

In the theoretical part, which comprises Sections 5 to 7, we evaluate the performance of the analytic approximation from Pykhtin (2004) for economic capital, using EC estimates from Monte Carlo simulation as a benchmark. Section 5 focuses on higly granular portfolios which are homogeneous on a sector level and in particular on the sensitivity of the results to the number of selected factors. Section 6 deals with portfolios with lower granularity and Section 7 introduces in addition PD heterogeneity on an exposure level.

Section 8 summarises and concludes.

2. Measuring Concentration Risk in a Multi-Factor Model

2.1. General Framework

We assume that every loan in a portfolio can be assigned to a different borrower so that the number of exposures or loans equals the number of borrowers. Each borrower *i* can uniquely be assigned to a single specific sector.[6] Let M denote the total number of borrowers or loans in the portfolio, M_s the number of borrowers or loans in sector s, and S the total number of sectors. Each exposure has the same relative portfolio weight of $w_{s,i} = 1/M$. Therefore, the weight of the aggregated sector exposure, w_s, always equals M_s/M.

Each sector is characterized by a unique default probability, p_s, and expected loss severity, μ_s. In practice these parameters can easily be estimated from exposure-weighted averages for each sector. The dependence structure between borrower defaults is driven by sector-dependent systematic risk factors which are usually correlated. Each risk factor can be uniquely assigned to a different sector, so that the number of sectors and factors are the same. The correlation structure is completely defined by sector-dependent asset correlations r_s between a borrower and a sector-dependent systematic factor and by pairwise (unconditional) correlations $\rho_{s,t}$ between systematic factors (often referred to as factor correlation).

The general framework is a multi-factor default-mode Merton-type model.[7] In this type of model, (often referred to as a variant of *asset-value models* or *structural models*), the unobservable, normalised asset return $X_{s,i}$ of borrower i in sector s triggers the default event. Credit risk occurs only as a default event which is consistent with traditional book-value accounting, the basis of traditional loan portfolio management.

The latent variable $X_{s,i}$ follows a factor model and can be written as a linear function of an industry sector risk factor Y_s and an idiosyncratic risk factor $\varepsilon_{s,i}$:

$$(1)\quad X_{s,i} = r_s Y_s + \sqrt{1-r_s^2}\,\varepsilon_{s,i}\,.$$

The sector factor weight r_s denotes the sensitivity of the asset returns of firms in sector s to the industry risk factor Y_s. The higher the value of the factor weight, the more sensitive the asset returns of firm i are to the sector factor. The disturbance term $\varepsilon_{i,k}$ follows a standard normal

6 In practice (large) firms often comprise business lines from different industry sectors. However, we pose this assumption here for practical and presentational purposes.

7 See also Gupton et al. (1997), Gordy (2000), and Bluhm et al. (2003) for more detailed information on these types of models. The origin of these models can be found in the seminal work by Merton (1974).

distribution. The sector factor weight also determines the factor weights of the idiosyncratic risk factor in order to retain a standard normal distribution for $X_{s,i}$.

The correlations between the systematic sector risk factors Y_s are denoted by $\rho_{s,t}$ and often referred to as factor correlations. The sector factors can be expressed as a linear combination of independent, standard normally distributed factors $Z_1, \ldots, Z_S$.

$$Y_s = \sum_{j=1}^{S} \alpha_{s,j} Z_j \text{ with } \sum_{j=1}^{S} \alpha_{s,j}^2 = 1 \text{ for } 1 \le s \le S.$$

The matrix $\left(\alpha_{s,t}\right)_{1 \le s,t \le S}$ is obtained from a Cholesky decomposition of the factor correlation matrix. The asset correlation for each pair of borrowers i and j in sectors s and t is then given by

$$(2) \qquad cor(X_{s,i}, X_{t,j}) = r_s \; r_t \; \rho_{s,t} = r_s \; r_t \sum_{n=1}^{S} \alpha_{s,n} \alpha_{t,n} \; .$$

Dependencies between borrowers arise only from their affiliation with the industry sector and from the correlations between the sectors factors.

Default occurs when the latent variable $X_{s,i}$ falls below a threshold that can be calculated as $N^{-1}(p_{s,i})$ if $p_{s,i}$ denotes the unconditional probability of default and $N^{-1}(\;)$ the inverse of the cumulative standard normal distribution function. If a firm defaults, the amount of loss is determined by the loss severity $\psi_{s,i}$. We assume that the loss severity is known at default and that before this event it is subject only to idiosyncratic risk. Credit losses of the whole portfolio are then given by

$$(3) \qquad L = \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{s,i} \psi_{s,i} 1_{\left\{X_{s,i} \le N^{-1}(p_{s,i})\right\}} \; .$$

In summary, the model needs the following input parameters:

- the relative exposure size $w_{s,i}$ of borrower i in sector s,
- the default probability $p_{s,i}$ of borrower i in sector s,
- the expected loss severity $\mu_{s,i}$ of borrower i in sector s
- the factor correlation matrix, and

- the factor loading r_s

In the following we assume that borrowers are homogeneous in each sector in terms of these parameters. Further, we assume that the factor correlation matrix can be proxied by a correlation matrix of equity index returns. In principle, asset correlations could be directly estimated from the time series of asset values. However, asset values are usually not observable. Since equity can be viewed as a call option on a firm's assets, it is frequently argued that correlation estimates from equity returns provide a viable approximation of factor correlations.[8]

2.2. The CreditMetrics Default-Mode Model

To obtain the loss distribution CreditMetrics applies Monte Carlo simulations by generating asset returns and counting the default events. In each simulation run the portfolio loss is determined from equation (3). For each exposure, the asset returns are determined and compared with the default threshold. The asset returns are calculated from equation (1). The sector factors Y_s and the individual risk contributions $\varepsilon_{s,i}$ are randomly sampled. If the value of the asset returns falls below the threshold $N^{-1}(p_{s,i})$, the borrower is in default. The portfolio loss of a simulation run is calculated by adding up the incurred losses from defaulted borrowers. The number of simulation runs in our analyses is typically 500,000. Portfolio losses obtained in each simulation run are sorted to form the distribution of portfolio losses from which several credit risk measures (such as VaR, EC,…) can be determined. In what follows, we will only report the economic capital, EC_{sim}, of the loan portfolio.

2.3. Analytic EC Approximation

In this section, we describe an analytical approximation to the VaR in the framework of a multi-factor model, which is a simplified version of the model developed by Pykhtin (2004) The main advantage of this model is its tractability since it does not require Monte Carlo simulations. Furthermore, we have simplified the model in such a way that it only requires exposure size, PD, and expected loss severities aggregated on a sector level instead of on an

8 Previous analyses (see, for example, Zeng and Zhang (2001) have shown that equity correlations may not be the best proxies for asset correlations given that equity correlations may be subject to noise which may not be necessarily related to firms' fundamentals. However, the main advantage of using equity data is that there is an abundance of data. Therefore, it is no surprise to see that it has become market practice to use equity correlation as a proxy for asset correlation.

exposure level. The factor correlation matrix and the factor loadings are still needed as in the CreditMetrics model.

Based on the work by Gouriéroux et al. (2000) and Martin and Wilde (2002) and not considering higher-order terms we can approximate the "true" loss L of a portfolio by a perturbed loss variable $L_\varepsilon = \bar{L} + \varepsilon\left(L - \bar{L}\right)$. $\bar{L}$ denotes the loss in the asymptotic single risk factor (ASRF) model with infinitely granular sectors and depends on the default probability $\hat{p}(\bar{Y})$ conditional on the systematic risk factor $\bar{Y}$

$$(4) \qquad \bar{L} = \sum_{s=1}^{S} w_s \mu_s \hat{p}_s\left(\bar{Y}\right) \text{ with } \hat{p}_s(\bar{Y}) = N\left(\frac{N^{-1}(p_s) - c_s \bar{Y}}{\sqrt{1-c_s^2}}\right).$$

The parameter c_s can be interpreted as the correlation bewteen the systematic factor returns and the normalised asset returns.The q-quantile of the "true" loss distribution, $t_q(L)$, can be approximated by $t_q\left(L_\varepsilon\right)$ as the sum of the VaR in the ASRF model , $t_q(\bar{L})$, and a multi-factor adjustment, $\Delta t_q \equiv t_q\left(L\right) - t_q\left(\bar{L}\right)$. This multi-factor adjustment can be determined by a second-order Taylor series expansion. The first-order effect vanishes because we require $\bar{L} = E\left[L \mid \bar{Y}\right]$. Neglecting higher-order terms, we can approximate $t_q(L)$ as follows:[9]

$$(5) \qquad t_q(L) \approx t_q(\bar{L}) + \frac{1}{2}\left.\frac{d^2 t_q(L_\varepsilon)}{d\varepsilon^2}\right|_{\varepsilon=0}$$

The first summand in (5) denotes the VaR in the single risk factor model and can be calculated by replacing $\bar{Y}$ in (4) by $t_q\left(\bar{Y}\right)$. If $\bar{Y}$ follows a standard normal distribution, $t_q\left(\bar{Y}\right)$ is the q-quantile of this distribution. Note that this single risk-factor model differs from the well-known ASRF model in that $\hat{p}_s\left(t_q\left(\bar{Y}\right)\right)$ depends on a sector-dependent asset correlation. To avoid confusion, we will call this model the "ASRF* model", reserving the term "ASRF model" for the model with uniform asset correlations.

The second summand in (5) denotes the multi-factor adjustment, Δt_q, which can be calculated according to Pykhtin (2004) by

$$(6) \qquad \Delta t_q = -\frac{1}{2l'(y)}\left.\left[v'(y) - v(y)\left(\frac{l''(y)}{l'(y)} + y\right)\right]\right|_{y=N^{-1}(1-q)}$$

[9] See Pykhtin (2004) for proofs.

where $l'(y)$ and $l''(y)$ denote, respectively, the second and first derivative of

$$l(y) = \sum_{s=1}^{N} w_s \mu_s \hat{p}_s(y) .$$

The remaining open issue is how to establish the link between L and $\bar{L}$. This is achieved by a linear mapping of the risk factors $Z_1, \ldots, Z_S$ to the single factor $\bar{Y}$,

$$\bar{Y} = \sum_{s=1}^{S} b_s Z_s .$$

The correlations in the multi-factor model are used to calculate the (also sector-dependent) correlations in the ASRF* model using the following mapping function, for $s \in \{1, \ldots, S\}$:

$$c_s = r_s \sum_{j=1}^{S} \alpha_{s,j} b_j .$$

There is no unique solution for determining the coefficients $b_1, \ldots, b_S$. In the following, we will use the approach in Pykhtin (2004) which is briefly summarised in Appendix C.

3. Portfolio Composition

3.1. Data set and sectoral definitions

Our analyses are based on loan portfolios which reflect characteristics of real bank portfolios, which were obtained from European credit register data. Our benchmark portfolio represents the overall sector concentration of the German banking system as it was constructed by aggregating the exposure values of loan portfolios of 2224 German banks in September 2004. The portfolio includes branches of foreign banks located in Germany. Credit exposures to foreign borrowers, however, are excluded. We deem this to be a reasonable approximation of a well-diversified portfolio based on the intuition that a portfolio cannot be more diversified than in the case in which it represents the average relative sector exposures of the national banking system. In principle, we could also have created a more diversified portfolio in the sense of having a lower VaR. However, such a portfolio would be specific to the credit risk model used and would not be obtainable for all banks.

All credit institutions in Germany are required by the German Banking Act (*Kreditwesengesetz*) to report quarterly exposure amounts of those borrowers whose indebtedness to them amounts to at least €1.5 million or more at any time during the three calendar months preceding the reporting date. In addition, banks report national codes that are compatible with the NACE classification scheme and indicate the economic activity of the borrower and his country of residence. Individual borrowers are summarised to *borrower units* which are linked, for example, by investments and constitute an entity sharing roughly the same risk. The aggregation of exposures on a business sector level was carried out on the basis of borrower units. If borrowers in that unit belong to different sectors, the dominating exposure amount determined the final sector allocation. Therefore, the credit register not only includes exposures above €1.5 million but also smaller exposures to individual borrowers belonging to a borrower unit that exceeds this exposure limit. This characteristic substantially increases its coverage of the credit market.

The industry classification chosen by CreditMetrics is the Global Industry Classification Standard (GICS), which was jointly launched by Standard & Poor's and Morgan Stanley Capital International (MSCI) in 1999. The classification scheme was developed to establish a global standard for categorising firms into sectors and industries according to their principal business activities. It comprises 10 broad sectors which are divided into 24 industry groups.[10] GICS further divides these groups into industries and sub-industries. However, the latter detailed schemes are not used by vendor models. In the following we use the broad sector classification scheme. Because some of the industry groups that form the broad sector Industrial are very heterogeneous, we decided to split this sector into the three industry groups: Capital Goods (including construction), Commercial Services / Supplies and Transportation.[11]

Credit register datasets, however, use the NACE industry classification system, which is quite different from the GICS system. In order to use the information from the credit register, we have performed a mapping[12] from the NACE codes to the GICS codes. A similar mapping is used by other vendor models, such as S&P’s Portfolio Risk Tracker developed by S&P. We have excluded exposures to financials (sector G) (which comprises exposures to banks (G1), diversified financials (G2), insurance companies (G3), and real estate (G4)). We have excluded exposures to the financial sector because of the specificities of this sector. Exposures to the real estate sector are heavily biased as it comprises a large number of exposures to borrowers that are related to the public sector. Since we could not differentiate between private and public enterprises in the real estate sector, we have exclude this sector from the following analyses. We also have disregarded exposures to households since it was impossible to find a

[10] See Table 12 in Appendix A, which shows the broad sectors and the more detailed industry groups.

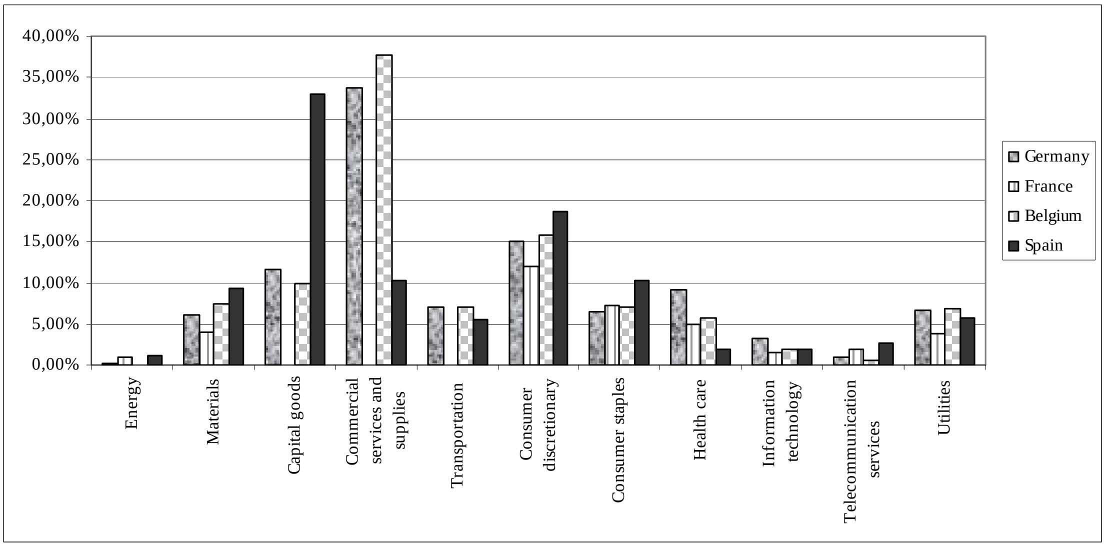
[11] Unreported simulations have shown that results are not affected when using the more detailed classification scheme.

representative equity index for them. This is a typical limitation of models relying on equity data for the estimation of asset correlations. In sum, we distinguish between 11 sectors, which can be considered as broadly representing the Basel II asset classes corporate and SME.

3.2. Comparison with French, Belgian and Spanish banking systems

A rough comparison of the relative share of the sector decomposition between the aggregated German, French, Belgian and Spanish banking systems shows that the numbers are relatively similar.[13] The only noticeable difference is the greater importance of the Capital Goods sector (33%) in Spain compared to Germany and Belgium and the smaller importance of the Commercial Services and Supplies Sector in Spain compared to Germany and Belgium. In general, however, the average sector concentrations are very similar across the four countries, which suggests that our results are to a large extent transferable to these countries.

Figure 1: Comparison of average sector concentration for Germany, France, Belgium, and Spain (*)



(*) A breakdown of the C sector for France is not available. The data on the aggregated C sector for France, however, is very similar the aggregated sector for Germany, Belgium and Spain.

3.3. Description of the benchmark portfolio

The sectoral distribution of exposures in the benchmark portfolio is shown in Table 1. assuming that the total portfolio has a volume of € 6 million. As mentioned above, this

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[12] See Table 13 in Appendix A for the mapping.

[13] The exact figures are provided by Table 14 in Appendix A.

portfolio represents the sectoral distribution of aggregate exposures in the German banking system. The degree of concentration in our reference portfolio is purely national and driven by the firms' sector composition because we exclude the impact of regional or country factors from our analysis. It is not uncommon that a bank uses a more detailed sector classification scheme. We consider it as more conservative to use a relatively broad sector classification scheme rather than a very detailed scheme. In a broad sector classification scheme, a larger proportion of exposures is attached to a sector. Therefore, correlations between exposures of the same sector, which are typically greater than the correlations between exposures of a different sector, will play a larger role.

In order to focus on the impact of sector concentration we assume an otherwise homogeneous portfolio by requiring that all other characteristics of the portfolio are uniform across sectors. We assume a total portfolio volume of € 6 million that consists of 6000 exposures of equal size and a uniform probability of default of 2%. Every exposure is to a different borrower, thus getting around the need to consider multiple exposure defaults. We set a uniform LGD of 45%, which is the corresponding supervisory value for a senior unsecured loan in the Foundation IRB approach of the revised Framework.[14] In the CreditMetrics approach industry weights can be assigned to each borrower according to its participation. Here, we assume that every firm is exposed to only one single sector as its main activity. Furthermore, we assume banks do not reduce exposure to certain sectors by purchasing credit protection.

Table 1: Composition of the benchmark portfolio (using the GICS sector classification scheme)

	Total exposure	Number of exposures	% exposure
A: Energy	11 000	11	0.18%
B: Materials	361 000	361	6.01%
C1: Capital Goods	692 000	692	11.53%
C2: Commercial services and supplies	2 020 000	2 020	33.69%
C3: Transportation	429 000	429	7.14%
D: Consumer discretionary	898 000	898	14.97%
E: Consumer staples	389 000	389	6.48%
F: Health care	545 000	545	9.09%
H: Information technology	192 000	192	3.20%
I: Telecommunication services	63 000	63	1.04%
J: Utilities	400 000	400	6.67%
Total	6 000 000	6 000	

3.4. Sequence of portfolios with increasing sector concentration

In order to measure the impact on *EC* of more concentrated portfolios than the benchmark portfolio, we construct a sequence of six portfolios, each with increased sector concentration relative to the previous portfolio in the sequence. To this end, we gradually increase sector

[14] See BCBS (2004b).

concentration in our benchmark portfolio by using the following algorithm. In each step we remove *x* exposures from all sectors and add them to a previously selected sector. This procedure is repeated until a single-sector portfolio is obtained, which is the portfolio with the highest possible concentration. The sector which receives *x* exposures in every step and also the amount *x* that is transferred to this sector are determined in such a way that some of the generated portfolios reflect a degree of sector concentration that is actually observable in real banks.[15]

Table 2 and Figure 2 show a sequence of seven portfolios, in order of increasing sector concentration. The increase in sector concentration is also reflected in the Herfindahl-Hirschmann-Index (HHI),[16] given in the last row which is calculated at sector level. Portfolio 1 has been constructed from the benchmark portfolio by re-allocating one third of each sector exposure to the sector Capital Goods. The even more concentrated portfolios 2, 3, 4 and 5 have been created by a repeated application of this rule. Portfolios 2 and 5 are similar to real portfolios of existing banks[17] insofar as the sector with the largest exposure size has a similar share of the total portfolio. Furthermore, the HHI is similar to what is observed in real-world portfolios. Finally, we created portfolio 6 with the highest degree of concentration as a one-sector portfolio by shifting all exposures to the Capital Goods sector.

Table 2: Sequence of portfolios with increasing sector concentration reflecting real portfolios

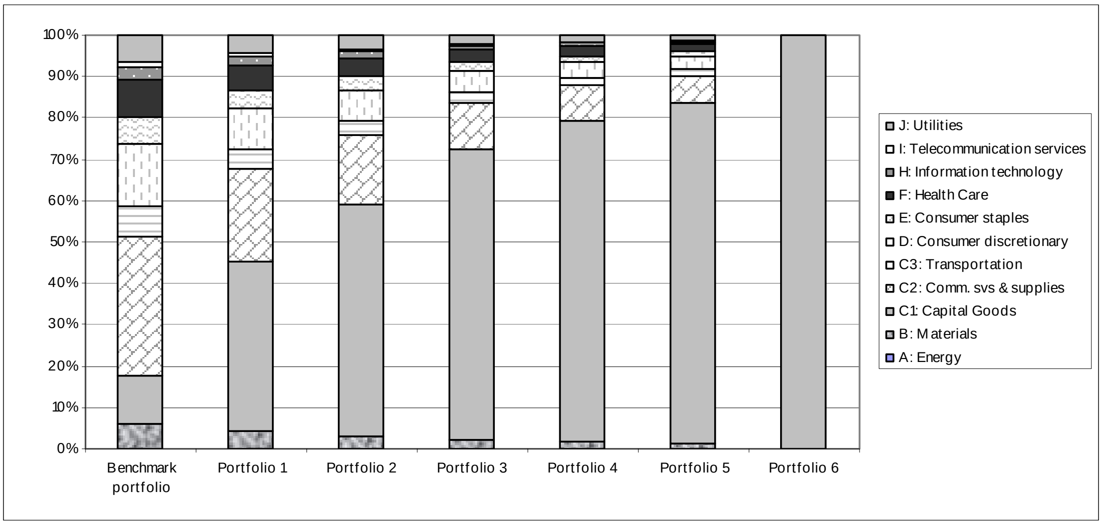
	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
A: Energy	0%	0%	0%	0%	0%	0%	0%
B: Materials	6%	4%	3%	2%	2%	1%	0%
C1: Capital gods	12%	41%	56%	71%	78%	82%	100%
C2: Comm. services & supplies	34%	22%	17%	11%	8%	7%	0%
C3: Transportation	7%	5%	4%	2%	2%	1%	0%
D: Consumer discretionary	15%	10%	7%	5%	4%	3%	0%
E: Consumer staples	6%	4%	3%	2%	2%	1%	0%
F: Health care	9%	6%	5%	3%	2%	2%	0%
H: Information technology	3%	2%	2%	1%	1%	1%	0%
I: Telecommunication services	1%	1%	1%	0%	0%	0%	0%
J: Utilities	7%	4%	3%	2%	2%	1%	0%
HHI	17.6	24.1	35.2	51.5	61.7	68.4	1

15 Due to a confidentiality requirements, we cannot reveal more detailed information.

16 See Hirschmann (1964).

17 Confidentiality requirements dictate that those banks with a high sector concentration must remain anonymous.

Figure 2: Sequence of portfolios with increasing sector concentration reflecting real portfolios



3.5. Intra- and inter- sectoral correlations

The sector factor correlations are estimated from historical equity index correlations. Table 2 shows the equity correlation matrix of the relevant MSCI EMU industry indices.[18] The sector factor correlations are based on weekly return data covering the period from November 2003 to November 2004. Sectors that are highly correlated with other sectors (i.e. sectors that have an average inter-sector equity correlation of greater than 65%) are: Materials (B), Capital Goods (C1), Transportation (C3) and Consumer Discretionary (D). Sectors that are moderately correlated with other sectors, i.e. sectors that have an average inter-sector equity correlation of between 45% and 65%, are Commercial Services and Supplies (C2), Consumer Staples (E) and Telecommunication (I). Sectors that are the least correlated with other sectors, i.e. sectors that have an average inter-sector equity correlation of less than 45%, are: Energy (A) and Health Care (F). The relative order of these sectors is broadly in line with results reported in other empirical papers.[19] The heterogeneity between the Capital Goods, Commercial Services and Supplies and Transportation sectors are confirmed by noticeable differences in correlations. The intra-sector correlations and/or inter-sector correlations between exposures are obtained by multiplying these sector factor correlations of Table 3 with the factor weights of the exposures.

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18 The correlation matrix based on MSCI US data is similar.

19 See, for example, De Servigny and Renault (2001), FitchRatings (2004) and Moody's (2004). It is difficult to compare the absolute inter-sector correlation values as different papers report different types of correlations. De Servigny and Renault (2001) report inter-sector default correlation values, FitchRatings (2004) reports inter-sector equity correlations while Moody's (2004) provides correlation estimates inferred from co-movements in ratings and asset correlation estimates. Furthermore, the different papers distinguish between a different number of sectors.

Table 3: Correlation matrix based on MSCI EMU industry indices (based on weekly log return data covering the Nov 2003 - Nov 2004 period; in percentages).

	A	**B**	**C1**	**C2**	**C3**	**D**	**E**	**F**	**H**	**I**	**J**
A: Energy	100	50	42	34	45	46	57	34	10	31	69
B: Materials		100	87	61	75	84	62	30	56	73	66
C1: Capital Goods			100	67	83	92	65	32	69	82	66
C2: Comm. svs & supplies				100	58	68	40	8	50	60	37
C3: Transportation					100	83	68	27	58	77	67
D: Consumer discretionary						100	76	21	69	81	66
E: Consumer staples							100	33	46	56	66
F: Health Care								100	15	24	46
H: Information technology									100	75	42
I: Telecommu-nication services										100	62
J: Utilities											100

We do not use the formula provided in CreditMetrics to determine the weights for the systematic risk factors, as recent research has suggested that this formula does not fit the German data very well.[20] Instead, we assume that all exposures are equally sensitive to the systematic factor. The value of the factor loading is calibrated to the corresponding IRB regulatory capital charge. More precisely, we determine a factor loading $r_s=0.50$ for all sectors $s \in \{1,...,S\}$, which ensures that the economic capital EC_{sim} equals the IRB capital charge for corporate exposures, assuming a default probability of 2%, an LGD of 45%, and a maturity of one year.

This value of the sector factor weight is slightly more conservative than empirical results for German companies suggest. The average of all the correlation entries in the factor correlation matrix is 0.59, which implies an average asset correlation between exposures of 0.14. Empirical evidence[21] has shown that German SMEs typically have an average asset correlation of 0.09, which implies that $r_s = 0.4$. Large firms, however, are typically more exposed to systematic risk than SMEs and therefore usually have higher asset correlation values.[22]

Intra-sector asset correlations between exposures are thus fixed at 25%. Inter-sector asset correlations between exposures can be calculated by multiplying the factor weights of both sectors by the inter-sector equity correlation. The lowest equity correlation between the Energy equity index and the Information Technology index of 10% translates into inter-sector asset correlations of 2.5%. The highest equity index correlation occurs between the Commercial Services and Supplies and the Consumer Discretionary sector index. At 92%, it translates into an inter-sector asset correlation of 23%.

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[20] See Hahnenstein (2004) for a detailed analysis.

[21] See Hahnenstein (2004).

[22] See, for example, Lopez (2004) for empirical evidence of this relation for the US.

4. Impact of sector concentration on economic capital

In this section we analyse the impact of increasing sector concentration on economic capital, which is defined as the difference between the 99.9% percentile of the loss distribution and the expected loss. The sults for the *EC* of the seven portfolios is given in Table 4. We observe for the corporate portfolios that economic capital increases from the benchmark portfolio to portfolio 2 by 20%. Economic capital for the concentrated portfolio 5 increases by a substantial 40% relative to the benchmark portfolio. These results demonstrate the importance of taking sector concentration into account when calculating *EC*.

Typically the corporate portfolio comprises only a fraction of the total loan portfolio (which also contains loans to sovereigns, other banks and private retail clients). Although the increase in sector concentration may have a significant impact on the economic capital for the corporate credit portfolio, it may have a much smaller impact in terms of a bank’s total credit portfolio. For a meaningful comparison, we assume that the corporate credit portfolio comprises 30% of the total portfolio and that the banks need to hold capital amounting to 8% of their total portfolio. By assuming that there are no diversification benefits between corporate exposures and the bank's other assets, the *EC* of the total portfolio can be determined as the sum of the *EC* for the corporate exposures and the EC for the remaining exposures.

Table 4: Impact of sector concentration on economic capital (EC_{sim}) for the sequence of corporate portfolios and for the sequence of total portfolios of a bank

	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Corporate portfolio	7.8	8.8	9.5	10.1	10.3	10.7	11.7
Total portfolio	8.0	8.2	8.5	8.7	8.8	8.9	9.2

The results for the total portfolios of the bank are also shown in Table 4. For the total portfolios 1 to 6 *EC* changes only because the sector concentration in the corporate portfolio increases, whereas *EC* for other assets remains constant. As expected, the impact of an increase in sector concentration is much less severe when looking at the *EC* for the total portfolio. We observe that total economic capital increases from the benchmark portfolio to portfolio 2 by 5%. Economic capital for the concentrated portfolio 5 increases by about 16% relative to the benchmark portfolio.

The procedure to generate a sequence of portfolios with increasing sector concentration is by no means unique. Therefore, we employ two alternative rules to generate these portfolios which are described in Appendix D. The key idea is that each new sequence of portfolios is generated by assigning exposures to the sector, which exhibits the highest (the ‘High-MEC rule’) or by assigning exposures to the sector with the lowest marginal economic capital (the ‘Low MEC rule’). The impact of an increasing sector concentration under these alternative rules of portfolio generation is presented in Table 17 in Appendix D. Economic capital increases significantly with sector concentration and, in a similar range also under these additional construction rules.

If we compare the increase in economic capital under the three construction rules, we see that it increases fastest for the sequence of portfolios which are generated by the "High-*MEC*"-rule. This is due to the fact that the Commercial Services and Supplies sector is more correlated with other sectors than the Capital Goods sector. For the same reasons, economic capital for the sequence of portfolios generated by the "Low-*MEC*"-rule increases at the slowest pace. The difference between economic capital under the three construction rules diminishes as sector concentration increases. When we increase the share of a sector in a portfolio, intra-sector asset correlations (which we have assumed to be the same for all sectors) play a larger role in determining economic capital. Since we have assumed that all characteristics of the portfolio are uniform across sectors, we see that in the one-sector case of portfolio 6 the results are the same under both rules.

In order to verify how robust our results are to the input parameters we have carried out the following four robustness checks (RC1 – RC4):

- a lower uniform PD of 0.5% instead of 2% for all sectors (RC1),
- heterogeneous PDs which were estimated from historical default rates of the individual sectors (RC2) and given in Table 5,
- a different factor correlation matrix representing the correlation matrix with the highest average annual correlation over the period 1997-2005,
- a uniform intra-sector asset correlation of 15% and a uniform inter-sector asset correlation of 6% (RC4), which are values also used by Moody's for the risk analysis of synthetic CDOs.[23]

[23] See Fu et al (2004).

Table 5: Average default rates (1990-2004)

Sector	Default rate
A: Energy	1.5%
B: Materials	2.8%
C1: Capital goods	2.9%
C2: Commercial services and supplies	3.7%
C3: Transportation	2.9%
D: Consumer discretionary	3.2%
E: Consumer staples	3.5%
F: Health care	1.6%
H: Information technology	2.4%
I: Telecommunication services	3.6%
J: Utilities	0.6%

Source: Own calculation, based on S&P (2004)

Although the absolute level of *EC* varies between these robustness checks, the relative increase in EC compared with the benchmark portfolio is similar to previous results in this section. The results are summarised in Table 6. For Moody's correlation assumptions in RC4, the increase in *EC* is stronger than for the other robustness checks. This can be explained by the bigger difference between intra-sector and inter-sector correlations, which is justified by their higher number of sectors they use, and which leads to a stronger *EC* increase when the portfolio becomes more and more concentrated in a single sector. We conclude that the observed substantial relative increase in *EC* due to introducing sector concentration is robust against realistic variation of the input parameters. Furthermore, this increase in *EC* may even be stronger dependent on the underlying dependence structure.

Table 6: EC in % of total exposure for the benchmark portfolio and its relative increase for the more highly concentrated portfolios 1 - 6

	Using "Real-rule"	RC1: PD=0.5%	RC2: Heterogeneous PD	RC3: Higher correlation	RC4: Moody's
EC					
Benchmark portfolio	7.8	3.3	10.0	8.7	4.0
Change of EC in %					
Portfolio 1	+13	+12	+11	+6	+6
Portfolio 2	+20	+21	+15	+13	+18
Portfolio 3	+30	+29	+25	+22	+39
Portfolio 4	+35	+37	+27	+24	+46
Portfolio 5	+36	+42	+32	+24	+51
Portfolio 6	+49	+52	+42	+33	+77

5. Evaluation of the EC Approximations for Homogeneous Sector-wide PDs and High Granularity

The purpose of this section is to analyse the performance of the *EC* approximations, given homogeneity within each sector and assuming a highly granular exposure distribution in each sector. Since these are two assumptions of our model, the results can be interpreted to hold for best case scenarios in terms of approximation quality. The analysis is performed by varying one by one the sector distributions, the factor correlations, the factor weights, the number of factors and the sector PD. The case of less granular portfolios and heterogeneous PDs on an exposure level is studied in Section 6.

We again assume a confidence level q of 99.9% and employ the following three risk measures (where $EL = \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{s,i} \psi_{s,i} p_{s,i}$):

- economic capital in the ASRF* model which is defined as $EC^* = t_{99.9\%}\left(\bar{L}\right) - EL$,
- economic capital based on the multi-factor adjustment,
  $EC_{MFA} = t_{99.9\%}\left(\bar{L}\right) + \Delta t_{99.9\%} - EL$, and
- economic capital based on Monte Carlo (MC) simulations, EC_{sim} .

Firstly, we present results for the benchmark portfolio and for the more concentrated portfolios 1 – 6 in Table 7 . Input factors are assumed to be the same as in Section 4.

Table 7: Comparison of EC^* , EC_{MFA} , and EC_{sim} for different exposure distributions across sectors with increasing sector concentration given a default probability of 2%

	EC^*	EC_{MFA}	EC_{sim}
Benchmark portfolio	7.8	7.9	7.8
Portfolio 1	8.7	8.8	8.8
Portfolio 2	9.4	9.4	9.5
Portfolio 3	10.1	10.1	10.1
Portfolio 4	10.5	10.5	10.3
Portfolio 5	10.7	10.7	10.7
Portfolio 6	11.6	11.6	11.7

The *EC* figures for the benchmark portfolio in Table 7 show that EC^* and EC_{MFA} provide extremely accurate proxies for EC_{sim} . This result suggests that in the studied examples the calculation of EC^* may, in practice, be sufficiently accurate for certain risk-management purposes. The four *EC* estimates for the more highly concentrated portfolios 1 – 6 indicate that economic capital increases as expected but that our results for the approximation performance of EC^* and EC_{MFA} still hold.

Secondly, we check whether our results differ when we vary the underlying correlation structure. To this end we calculate the three risk measures, presented in Table 8, for different factor correlation matrices. More specifically, we assume homogeneous factor correlation matrices in which the entries (outside the main diagonal) vary between 0 and 1 in increments of 0.2. The last case, in which all factor correlations are equal to one, corresponds to the case of a single-factor model.

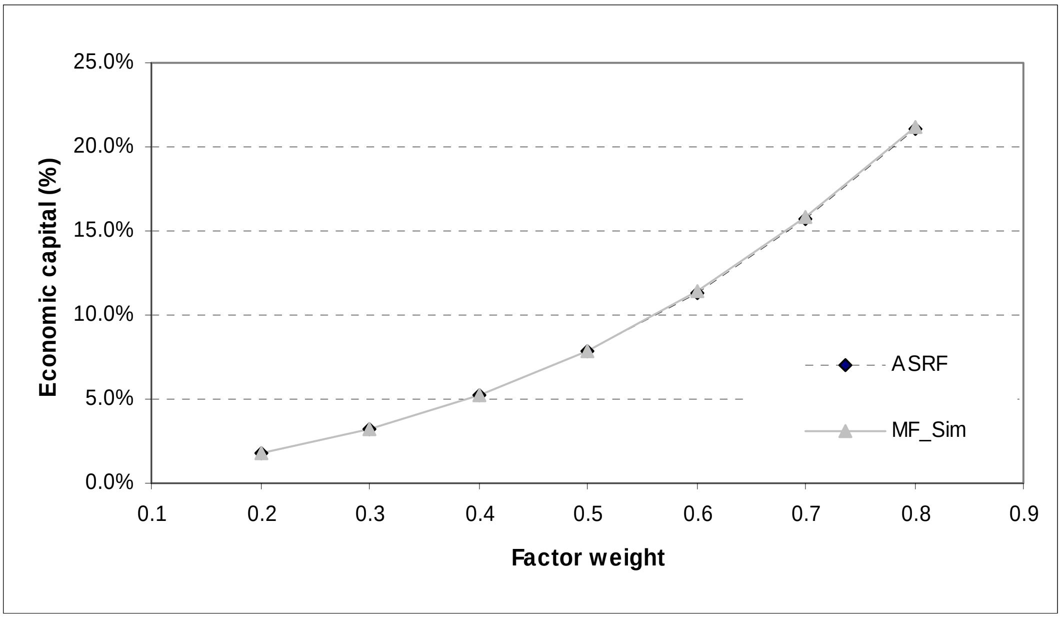
Table 8: Comparison of EC^*, EC_{MFA}, and EC_{sim} for different factor correlations ρ, given a default probability of 2%

ρ	EC^*	EC_{MFA}	EC_{sim}
0.0	3.3	3.9	4.0
0.2	4.5	4.9	5.0
0.4	6.1	6.3	6.3
0.6	7.9	7.8	8.0
0.8	9.7	9.7	9.9
1.0	11.6	11.6	11.9

Table 8 shows EC_{sim} and its proxies EC^* and EC_{MFA} for increasing factor correlations. As expected, economic capital increases with increasing factor correlations since a higher factor correlation reduces the diversification potential by shifting probability mass to the tail of the loss distribution. For a realistic factor correlation of 0.4 and higher, the relative error of EC_{MFA} is below one percent. The approximation accuracy increases with the asset correlation and the maximum relative difference between EC_{MFA} and EC_{sim} is only 3% and occurs for the (unrealistic, in practice) case of independent sectors. Higher factor correlations bring the multi-factor model closer to a one-factor model for which EC^* and EC_{MFA} coincide. Again, EC^* is relatively close to the *EC* values based on the MFA. Therefore, our earlier results concerning the approximation performance of EC^* and EC_{MFA} also hold under different factor correlation assumptions.

Thirdly, we relax the assumption that the factor weight *r* is fixed. Figure 4 shows *EC* obtained from simulation and its approximation from the ASRF* model for *r* varying between 0.2 and 0.8. *EC* strongly increases with the factor weight but this does not affect the approximation quality of EC^*. The values of EC_{MFA} are not shown in Figure 3 since they are even closer to *EC* and would be indistinguishable from the simulation estimate.

Figure 4: Comparison of EC^* *and* EC_{sim} *for different factor weights r, given a factor correlation of 0.6 and a default probability of 2%*



Fourthly, we explore how the results depend on the number of factors. To this purpose we vary the number of factors from 2 to 16. Figure 5 shows how EC^* and EC_{MFA} vary for different numbers of sectors and for different factor correlation values. In general we see that EC^* and EC_{MFA} decrease when the number of sectors increases. This result can be explained by diversification across sectors. In the single-factor case (factor correlation equals one) economic capital does not depend on the number of sectors any more. Figure 5 also shows that the approximation quality of EC^* increases with the number of sectors, which suggests to be careful when using EC^* for risk management purposes given a small number of factors.

Figure 5: Economic capital (EC^* and EC_{MFA}) for different factor correlation values for 2, 6, and 16 sectors

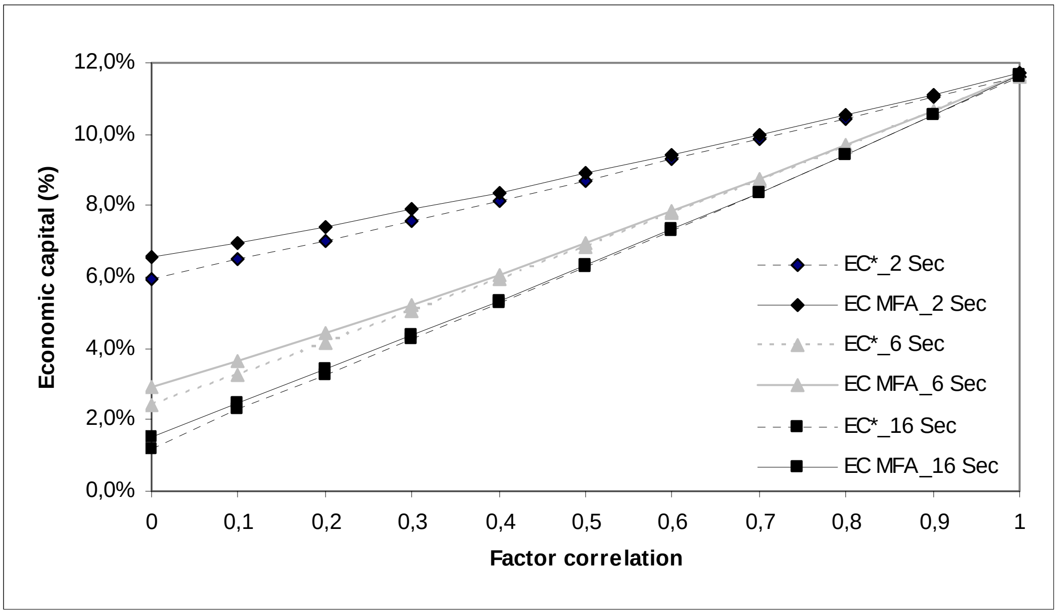
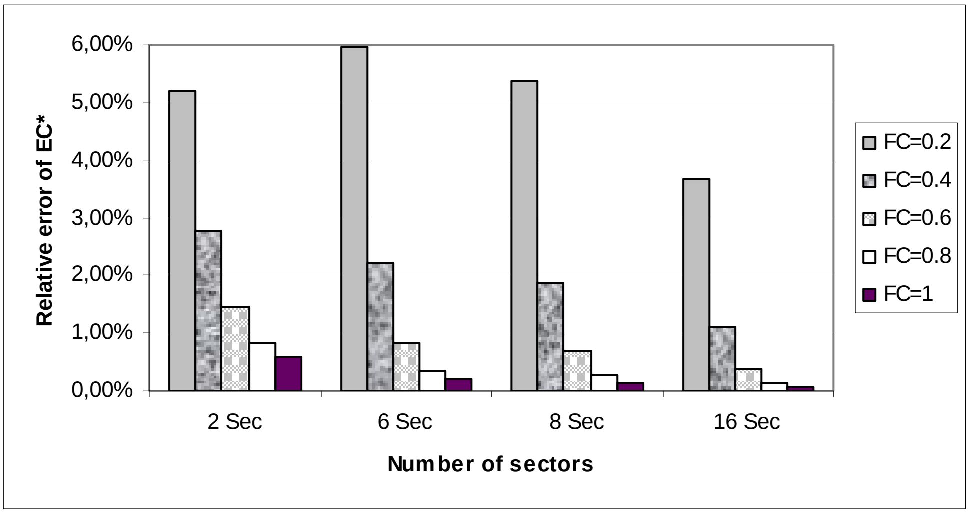


Figure 6 shows the relative measurement error of using EC^* instead of EC_{MFA} in the 2-, 6-, 8- and a 16-factor model in more detail. It confirms that in general the approximation error decreases with the number of factors. The multi-factor adjustment becomes more important for a coarser sector distribution. Consistent with previous analyses, the approximation error in all four factor models decreases when factor correlations increase from 0.2 to 1.

Figure 6: Measurement error of EC^* relative to EC_{MFA} in a 2-, 6-, 8-, and 16-sector model and for selected values of the factor correlation (FC)



Fifthly, we tested whether our results for the approximation performance of EC^* and EC_{MFA} are sensitive to the values of the PDs. To this end, we used the average sector-dependent default rates listed in Table 5 instead of a uniform PD for all sectors. The results are given in Table 9.

Table 9: Comparison of EC^*, EC_{MFA}, and EC_{sim}, based on sector-dependent default probabilities, estimated from historical default rates

	EC^*	EC_{MFA}	EC_{sim}
Benchmark portfolio	9.9	10.0	10.1
Portfolio 1	10.9	11.0	11.2
Portfolio 2	11.6	11.7	11.8
Portfolio 3	12.3	12.6	12.8
Portfolio 4	12.8	13.0	13.1
Portfolio 5	13.0	13.2	13.2
Portfolio 6	14.1	14.1	14.2

The results in Table 9 show for all risk measures an increase in *EC* relative to the numbers in Table 7 which is explained by the fact that the PDs are on average higher than the uniform value of 2% in Table 7. The approximation quality is a bit lower for the benchmark portfolio as the measurement errors of EC^* and EC_{MFA} increase relative to EC_{sim} to 2% and 1.4% respectively. However, the approximation quality decreases for the more concentrated portfolios 1 – 6. We conclude that the results obtained for the case of a uniform PD qualitatively also hold for the case of heterogeneous sector-PDs.

6. Evaluation of the EC Approximations for Homogeneous Sector-wide PDs and Low Granularity

Our previous simulation results which reveal a reasonably good approximation quality for EC^* and EC_{MFA} are obtained conditional on a uniform PD in every sector and highly granular portfolios since each individual exposure has a relatively small share of 0.017% (=1/6000) of the total portfolio volume. Portfolios in particular of small banks, however are not perfectly granular. In the following we explore the impact of lower granularity. From the set of seven portfolios only the benchmark portfolio and portfolio 6 are considered since they have the lowest and the highest sector concentration.

In order to obtain an upper bound of the potential impact of granularity we consider the highest individual exposure share that is admissible under the EC large exposure rules[24]. According to these rules an exposure is considered as "large" if its amount requires 10% or more of the regulatory capital. Further, banks are not allowed to have an exposure that

[24] See Directive 93/6/EEC of 15 March 1993 on the capital adequacy of investment firms and credit institutions..

requires at least 25% of the regulatory capital. Furthermore, the sum of all large exposures must not require more than 8 times the regulatory capital.

We assume that a bank's regulatory capital is 8% of its total loan volume. For a total portfolio value of 6000 currency units banks are required to hold 480 currency units in capital. Each large exposure requires a minimum amount of capital of 48 currency units and a maximum amount of 120 currency units. The total sum of all large exposures must not exceed 3840 currency units. With these restrictions, the least granular, admissable exposure distribution of our portfolio consists of

- 3840/120=32 loans of 120 currency units,
- 2160/47= 45 loan exposures of 47 currency units (which are just below the large exposure limit of 48), and
- a remaining single exposure of 45 currency units.

The sector distribution of the portfolio is shown in Table 19 in the Appendix.

Table 10: Economic capital for least granular single-sector portfolio under European large exposure rules in percent

Portfolio	EC^*	EC_{MFA}	EC_{sim}	Relative error of EC_{MFA}
Benchmark portfolio	7.8	7.9	9.1	-13%
Single sector portfolio	11.6	11.7	12.7	-8%

Simulated economic capital, EC_{sim}, and its proxies EC^* and EC_{MFA} are given in Table 10. EC_{sim} for the low granular benchmark portfolio is 1.3 percentage points or 17% higher than for the highly granular benchmark portfolio in Table 7. This difference appears to be substantial but we have to consider that the granularity of the portfolio in Table 10 is very low since it reflects the lowest granularity permissible under European bank regulation. The EC_{sim} measure for the single sector portfolio 6 in Table 10 is higher than for the benchmark portfolio, which is consistent with earlier reported results.

For the purpose of this analysis more important than the level of *EC* are the approximation errors of the *EC* proxies, EC^* and EC_{MFA}. The *EC* proxies are based on the assumptions of infinite granularity in each sector, while the EC_{sim} calculations take into account the PD heterogeneity and the granularity. The last column of Table 10 shows the relative error when EC_{MFA} is used instead of EC_{sim}. The negative sign of the relative errors signals that economic capital is underestimated by the analytic approximations. We conclude that the *EC* proxies can substanstially underestimate *EC* in the case of portfolios with extremely low granularity.

7. Evaluation of Pykhtin's Approximation Formula for Heterogeneous Sectors

In the following we explore the impact of heterogeneous PDs inside a sector in addition to the impact of low granularity. In order to study the impact of heterogeneity in PDs we differentiate between two levels, firstly, a *sector level* where PDs can only differ across sectors and, secondly, a (single) *exposure level*, where PDs can also differ in the same sector across exposures. So far we only considered PD variation on a sector level but not on the exposure level.

PD heterogeneity on a *sector level* is accounted for by using the historical default rates for sectors in Table 5. These default rates are on average higher than the value of 2% which is used for the PDs in the case of homogeneous PDs. In order to avoid a distortion of a comparison between the homogeneous and the heterogeneous case from this level effect we scale the historical default rate, p_s^{hist}, for every sector s as follows,

$$(7) \qquad p_s^{scaled} = p_s^{hist} \frac{0.02}{\sum_{s=1}^{S} w_s \cdot p_s^{hist}}.$$

In this way we ensure that the weighted average PD of the total portfolio stays at 2% also in the case of PD heterogeneity across sectors. This is reasonable since the important information that we want to explore from a heterogeneous PD distribution concerns the relative distribution of PDs and not their level.

In the case of PD heterogeneity on an *exposure level* different PDs need to be assigned in a meaningful way to single exposures in every sector. For this purpose we employ an empirical PD distribution, based on a balance sheet database, maintained by the Deutsche Bundesbank, which comprises firms which are eligible for submitting fine trade bills.[25] The PD distribution is given in Table 22 in the Appendix.

We distinguish between a sorted and an unsorted case. In the sorted case, we first *sort* exposures by decreasing size in every sector (see the portfolio in Table 19 and the *sorted*

[25] More details on the database and the logit-model that was used to determine the PDs can be found in Krüger et al. (2005).

portfolio of Table 21). We assign the high PDs to the smallest exposures and the low PDs to the largest exposures, which generates a negative correlation between exposure size and PD. A negative correlation emerged as a stylised fact in recent empirical literature.[26] In the *unsorted* case, the PDs are randomly allocated to the exposures within the sector. In these cases there is no systematic relation between exposure size and PD.

The results for PD heterogeneity in every sector are given in Table 11. The last column shows that the approximation error is lower when using heterogeneous PDs on the exposure level. In the one sector portfolio, the *EC* proxies turn positive implying that the ASRF* model actually overestimates *EC*. Overall, the approximation errors vary between between -6% and +9%.

Table 11: Economic capital for least granular single-sector portfolio under European large exposure rules in percent, empirical PD distribution on exposure level

Portfolio	***Sector PD***	***Exposure PD***	EC^*	EC_{MFA}	EC_{sim}	***Relative error of*** EC_{MFA}
Benchmark portfolio	Heterogeneous	Heterogeneous, sorted	7.7	7.8	8.3	-6%
		Heterogeneous, unsorted	7.7	7.8	8.4	-7%
Single sector portfolio	-	Heterogeneous, sorted	11.6	11.6	10.6	+9%
		Heterogeneous, unsorted	11.6	11.6	10.8	+7%

The reduction of economic capital due to heterogeneity on the exposure level has also been noted by Hanson et al. (2005) and can be explained by two effects. First, the reduction of EC_{sim} is due to the fact that a finer rating system generally provides a reduction in *EC* because of the concavity of *EC's* dependence on PD. Second, for the sorted portfolios there is an additional effect by construction which stems from the positive correlation between exposure size and PD.

Both effects counterbalance the general underestimation of the ASRF* model so that the approximation error, measured relative to EC_{sim}, is notably lower than in the case of homogeneous PDs in each sector. Further this effect counterbalances the effect of low granularity.

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[26] See, for example, Dietsch and Petey (2002) or Lopez (2004).

In summary, this analysis has shown that PD heterogeneity on the single exposure improves the performance of the analytic *EC* approximations because it reduces their structurally induced underestimation of *EC*. This effect is amplified if larger exposures or firms have lower PDs than smaller ones and in this case we even observed cases in which the ASRF* model overestimated *EC*. Furthermore, PD heterogeneity appears not to affect the relative difference between EC_{MFA} and EC^*.

8. Summary and Conclusions

The minimum capital requirements for credit risk in the IRB approach of Basel II implicitly assume that banks' portfolios are well diversified across business sectors. Potential concentration risk in certain business sectors is covered by Pillar 2 of the Basel II Framework which comprises the supervisory review process.[27] To what extent the regulatory minimum capital requirements can understate the required capital is an empirical question. In this paper we approached this question by using data from the German central credit register. Credit risk is measured by economic capital in a multi-factor asset value model by Monte Carlo simulations.

In order to measure the impact of concentration risk on economic capital we start with a benchmark portfolio that reflects average sector exposures of the German banking system. Since the exposure distributions across business sectors are similar in Belgium, France, and Spain, we expect that our main results also hold for other European countries. Starting with the benchmark portfolio, we have successively increased sector concentration in six steps, considering degrees of sector concentration which are observable in real banks. The last and most concentrated portfolio contained only exposures to a single sector. Compared with the benchmark portfolio, economic capital for the concentrated portfolios can increase by almost 37% and is even higher in the case of a one-sector portfolio. Under the assumption that the corporate credit portfolio comprises 30% of the total portfolio, economic capital for the total portfolio resembling a real portfolio increases by 11% relative to the corporate portfolio. This result clearly underlines the necessity to take inter-sector dependency into account for the measurement of credit risk. We subjected our results to various robustness checks. We found that the increase in economic capital may even be stronger, depending on the underlying dependence structure.

[27] See BCBS (2004b), paragraphs 770-777.

Recognising that concentration in business sectors can substantially increase economic capital raises the question of whether this risk can be approximated by a tractable and robust method that avoids the use of computationally burdensome Monte Carlo simulations. Therefore, we have evaluated the accuracy of a model developed by Pykhtin (2004) which provides an analytical approximation to economic capital in a multi-factor framework. We have applied a simplified, more tractable version of the model which requires the input parameters exposure size, PD, and expected loss severity only aggregated on a sector level instead of on an individual exposure level. In addition the model requires information on sensitivity to the systematic risk factor and on the correlation matrix of sector factors. Furthermore, we have evaluated to which extent EC^* as the first of two components of the analytic approximation of economic capital already provides a reasonable proxy.

We have shown that for portfolios with relatively granular and homogeneous sectors the analytic approximation formulae are slightly downward biased but they perform extremely well for portfolios with different sector concentrations and under various factor weights and correlation assumptions. Furthermore, we have found that EC^* is relatively close to the true economic capital values obtained by simulations arguably for most of the consisdered realistic input parameter tupels.

When we reduce granularity and introduce PD heterogeneity on the single exposure level this has two counterbalacing effects on the performance of the analytic approximations for economic capital. The reduction of granularity has the effect that the downward bias increases since economic capital increases without effecting the analytic approximations. In extreme cases of portfolios with the lowest granularity permissible by EU large exposure rules the downward bias increases to between -8% and -13%, dependent on the sector structure of the portfolio.

However, when heterogeneous PDs on the individual exposure level are introduced, this reduces economic capital compared to the case of homogeneous PDs inside every sector. However, it doesn't effect the analytic approximations which are based on average PDs. As a consequence the downward bias decreases. If exposure size and PD are negatively correlated, which is a widely held belief by practitioners and also in the literature,[28] economic capital is further reduced to the extent that in certain cases the analytic approximations can even overstate economic capital. The relative error of the analytic approximation relative to the true economic capital lies in a range between –6% and +9%. In summary we find that heterogenity in individual PDs and low granularity partly balance each other in their impact on the performance of the analytic approximations. Which effect prevails depends on the specific input parameters.

[28] See, for example, Lopez (2004) or Dietsch and Petey (2002).

In the cases under study it is possible to use the simplified version of the model without sacrificing much accuracy. This is an important result since it suggests that supervisors and banks can reasonably well approximate their economic capital for a typical loan portfolio without running computationally intensive Monte Carlo simulations.

Further research seems to be warranted, particularly in further advancing Pykhtin's methodology in a direction which improves its approximation accuracy without extending the data requirements. This could be achieved, for example, by exploring alternative ways to map the correlation matrix of the multi-factor model into sector-dependent asset correlations.

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Appendix A

Table 12: GICS Classification Scheme: Broad Sector and Industry Groups

A: Energy	A1: Energy
B: Materials	B1: Materials
C: Industrial	C1: Capital goods C2: Commercial services and supplies C3: Transportation
D: Consumer discretionary	D1: Automobiles and components D2: Consumer durables ans apparel D3: Hotels, restaurants and leisure D4: Media D5: Retailing
E: Consumer staples	E1: Food and drug retailing E2: Food, beverage and tobacco E3: Household and personal products
F: Health care	F1: Health care equipment and services F2: Pharmaceuticals and biotechnology
G: Financials	G1: Banks G2: Diversified financials G3: Insurance G4: Real estate
H: Information technology	H1: Software and services H2: Technology hardware & equipment H3: Semiconductors & semiconductor equipment
I: Telecommunication services	I1: Telecommunication services
J: Utilities	J1: Utilities

Table 13: Mapping from NACE codes to GICS codes

2 (or more) -digit code	Description	Mapped to GICS
1	Agriculture and hunting	E
2	Forestry	B
5	Fishing	E
10	Coal mining	B
11	Crude petroleum and natural gas extraction	A
12	Mining of uranium and thorium	B
13	Mining of metal ores	B
14	Other mining and quarrying	B
15	Food and beverages manufacturing	E
16	Tobacco manufacturing	E
17	Textile manufacturing	D
18	Textile products manufacturing	D
19	Leather and leather products manufacturing	D
20	Wood products	D
21	Pulp, paper and paper products	B
22	Publishing and printing	C2
23	Manufacture of coke, refined petroleum products and nuclear fuel	A
24 (excl 244)	Chemicals and chemical products manufacturing	B

244	Pharmaceuticals	F
25	Rubber and plastic manufacturing	D
26	Other non-metallic mineral products	B
27	Basic metals manufacturing	B
28	Fabricated metal manufacturing	B
29	Machinery and equipment manufacturing	C1
30	Office machinery and computers manufacturing	H
31	Electrical machinery manufacturing	H
32	TV and communication equipment manufacturing	H
33	Medical and optical instruments manufacturing	F
34	Car manufacturing	D
35	Other transport equipment manufacturing	D
36	Furniture manufacturing	D
37	Recycling	J
40	Gas and electricity supply	J
41	Water supply	J
45	Construction	C1
50	Car sales, maintenance and repairs	D
51	Wholesale trade	C2
52 (excl 5211, 522,523)	Retail trade	D
522, 523	Consumer staples	E
55	Hotels and restaurants	D
60	Land transport	C3
61	Water transport	C3
62	Air transport	C3
63	Transport supporting activities and travel agencies	C3
64	Post and telecommunication	I
65	Financial institutions	G1
66	Insurance	G3
67	Support to financial institutions	G1
70	Real estate	G4
71	Machinery and equipment leasing manufacturing	C1
72	Computer and related activities	H
85	Health care and social work	F
90	Sewage and refuse disposal	J
96	Residential property management	G4

Table 14: Comparison of sector concentrations, aggregated exposure values over banks in Germany, France, Belgium and Spain

Sector	Germany	France	Belgium	Spain
A1: Energy	0.18%	0.88%	0.05%	1,05%
B1: Materials	6.01%	3.97%	7.45%	9,34%
C: Industrial[29]	52.36%	63.82%	54.77%	48,53%
C1: Capital goods	11.53%		9.89%	32,90%
C2: Commercial services and supplies	33.69%		37.74%	10,20%
C3: Transportation	7.14%		7.14%	5,43%
D: Consumer discretionary	14.97%	11.91%	15.77%	18,60%
E: Consumer staples	6.48%	7.21%	7.05%	10,20%
F: Health care	9.09%	5.00%	5.64%	1,85%
H1: Software en services	3.20%	1.47%	1.86%	1,99%
I1: Telecommunication services	1.04%	1.91%	0.54%	2,67%
J1: Utilities	6.67%	3.82%	6.87%	5,77%

Table 15: Correlation matrix based on MSCI EMU industry indices (based on weekly log return data covering the Nov 2002 - Nov 2003 period; in percentages). The mean and standard deviation have been calculated as the average and the standard deviations of the different inter-sector equity correlations

	A	**B**	**C1**	**C2**	**C3**	**D**	**E**	**F**	**H**	**I**	**J**	Mean	Stdev
A: Energy	100	62	66	43	62	67	78	70	50	47	72	62	11
B: Materials		100	91	78	77	85	73	69	74	68	69	75	9
C1: Capital Goods			100	76	80	92	74	68	81	72	75	78	9
C2: Comm. svs & supplies				100	66	81	58	53	71	58	52	63	13
C3: Transportation					100	78	68	59	70	65	64	69	7
D: Consumer discretionary						100	71	66	86	72	70	77	9
E: Consumer staples							100	75	62	60	70	69	7
F: Health Care								100	55	44	70	63	10
H: Information technology									100	69	58	68	11
I: Telecommu-nication services										100	67	62	10
J: Utilities											100	67	7

[29] Aggregate of C1, C2 and C3 only used for comparison with French data. Not used in the analysis.

Appendix B

The multi-factor adjustment (MFA) is defined as $\Delta t_q \equiv t_q(L) - t_q(\bar{L})$ and can be calculated as follows:

$$\text{(A1)} \qquad \Delta t_q = -\frac{1}{2l'(y)}\left[v'(y) - v(y)\left(\frac{l''(y)}{l'(y)} + y \right) \right]\Bigg|_{y=N^{-1}(1-q)}$$

where y denotes the single systematic risk factor.

The first and second derivatives of the loss distribution function in a one-factor model are

$$\text{(A2)} \qquad \begin{aligned} l'(y) &= \sum_{s=1}^{N} w_s \mu_s \hat{p}_s'(y) \\ l''(y) &= \sum_{s=1}^{N} w_s \mu_s \hat{p}_s''(y) \end{aligned}$$

where $\hat{p}_s^{'}(y)$ and $\hat{p}_s^{''}(y)$ are, respectively, the first and the second derivatives of the conditional probability of default.

$$\text{(A3)} \qquad \begin{aligned} \hat{p}_s'(y) &= -\frac{c_s}{\sqrt{1-c_s^2}} N'\left(\frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} \right) \\ \hat{p}_s''(y) &= -\frac{c_s}{\sqrt{1-c_s^2}} \frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} N'\left(\frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} \right) \end{aligned}.$$

c_s is the effective factor loading which can be written as $c_s = r_s \bar{\rho}_s$ where $\bar{\rho}_s$ denotes the correlation between the composite sector factor Y_s and the systematic factor in the ASRF $\bar{Y}$.

The conditional variance $v(y) \equiv \mathrm{var}(L \mid \bar{Y} = y)$

$$v\ (y)=\sum_{s=1}^{S}\sum_{t=1}^{S}w_s w_t \mu_s \mu_t\left[N_2\left(N^{-1}\left(\hat{p}_s(y)\right),N^{-1}\left(\hat{p}_t(y)\right),\rho_{st}^{Y}\right)-\hat{p}_s(y)\hat{p}_t(y)\right]$$
$$+\sum_{s=1}^{S}w_s^2\mu_s^2\left[\hat{p}_s(y)-N_2\left(N^{-1}\left(\hat{p}_s(y)\right),N^{-1}\left(\hat{p}_s(y)\right),\rho_{ss}^{Y}\right)\right]$$
$$v'(y)=\sum_{s=1}^{S}\sum_{t=1}^{S}w_s w_t \mu_s \mu_t \hat{p}_s'(y)\left[N\left(\frac{N^{-1}\left(\hat{p}_t(y)\right)-\rho_{st}^{Y}\hat{p}_s(y)}{\sqrt{1-\left(\rho_{st}^{Y}\right)^2}}\right)-\hat{p}_t(y)\right]$$
$$+\sum_{s=1}^{S}w_s^2\mu_s^2\hat{p}_s'(y)\left[1-2\cdot N\left(\sqrt{\frac{1-r_s^2}{1+r_s^2}}N^{-1}\left(\hat{p}_s(y)\right)\right)\right]$$

where $N_2(\ )$ denotes the cumulative distribution function of the bivariate-normal distribution and ρ_{st}^{Y} has the meaning of a conditional asset correlation for two exposures in sectors *t* and *s*, conditional on $\overline{Y}$. This conditional asset correlation can be written as

$$\rho_{st}^{Y}=\frac{r_s r_t \rho_{st}-c_s c_t}{\sqrt{\left(1-c_s^2\right)\left(1-c_t^2\right)}}\ .$$

Appendix C

In Pykhtin (2004) the coefficients $b_1, ..., b_S$ are obtained by maximising the correlation between $\overline{Y}$ and the risk factors $Y_{1,...,}Y_S$ which leads to the following optimisation problem:

$$\max_{b_1,...,b_S} \sum_{s=1}^{S} \gamma_s \sum_{j=1}^{S} \alpha_{s,j} b_j \ .$$

subject to $\sum_{s=1}^{S} b_s^2 = 1$. The solution of this optimisation problem is given by

$$b_j = \sum_{s=1}^{S} \frac{\gamma_s}{\lambda} \alpha_{s,j} \ .$$

λ is the Lagrange multiplier chosen to satisfy the constraint. Again there is no unique solution for γ_s. We follow Pykhtin who reported good results when defining

$$\gamma_s = w_s \mu_s N\left(\frac{N^{-1}(p_s) - r_s N^{-1}(q)}{\sqrt{1 - r_s^2}} \right).$$

Appendix D

The first rule chooses the sector that receives in every step x exposures from the others as the one with the highest marginal economic capital (MEC_i) that is required if an additional exposure is added to this sector ("High-*MEC*"-rule). The other rule chooses the sector that receives in every step x exposures from the others as the one with the lowest MEC_i that is required if an additional exposure is added to this sector ("Low-*MEC*"-rule). The MEC_i is defined as the difference between the economic capital of the whole portfolio after one additional exposure unit has been added to sector i and the economic capital of the portfolio before. The MEC_i of all exposures sum up to the economic capital for the whole portfolio. By construction, the economic capital for another sequence of portfolios constructed by using a rule that concentrates the portfolio towards another sector will lay between the EC obtained using the "high-*MEC*"-rule and the "low-*MEC*"-rule.

Table 16 gives an overview of the MEC_i of the different sectors for the benchmark portfolio. The sector with the highest MEC_i appears to be Commercial services and supplies. This is an intuitive result because this sector is not only a large sector (so intra-sector correlation will play an important role) but it is also a sector which is moderately correlated with other sectors. The sector with the lowest MEC_i is the Energy sector which is a small sector that is one of the least correlated with other sectors.

Table 16: Marginal economic capital (benchmark portfolio intra-sector asset correlation 25%, inter-sector asset correlation 2.6% ≤ ρ_a≤23%)

	$MEC_{(i)}$ (q=99.9%) absolute values
A: Energy	5
B: Materials	29
C1: Capital Goods	54
C2: Comm. services & supplies	188
C3: Transportation	60
D: Consumer discretionary	52
E: Consumer staples	20
F: Health Care	13
H: Information technology	34
I: Telecommunication services	18
J: Utilities	24

Table 17 and Figure 7 present the sequence of portfolios in which exposures are transferred in every step towards the sector with the highest MEC_i (Commercial Services and Supplies).

Table 17: Sequence of portfolios with increasing sector concentration towards the sector with the highest MEC_i

	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
A: Energy	0%	0%	0%	0%	0%	0%	0%
B: Materials	6%	4%	3%	2%	2%	1%	0%
C1: Capital Goods	12%	8%	6%	4%	3%	2%	0%
C2: Comm. svs & supplies	34%	56%	67%	78%	83%	87%	100%
C3: Transportation	7%	5%	4%	2%	2%	1%	0%
D: Consumer discretionary	15%	10%	7%	5%	4%	3%	0%
E: Consumer staples	6%	4%	3%	2%	2%	1%	0%
F: Health Care	9%	6%	5%	3%	2%	2%	0%
H: Information technology	3%	2%	2%	1%	1%	1%	0%
I: Telecommunication services	1%	1%	1%	0%	0%	0%	0%
J: Utilities	7%	4%	3%	2%	2%	1%	0%
HHI	17.6	19	29.5	46.5	57.4	64.8	1

Figure 7: Sequence of portfolios with increasing sector concentration towards sector with highest MEC(i)

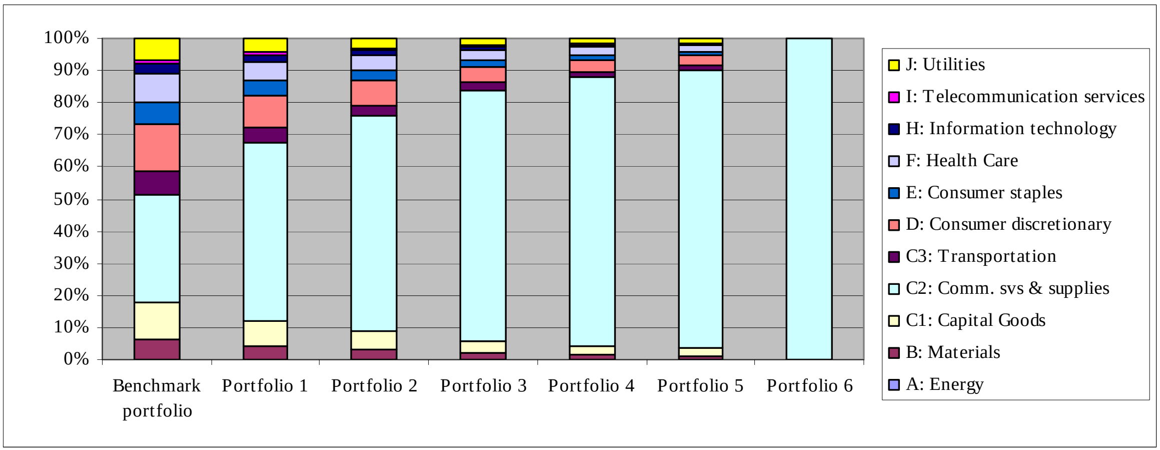

Table 18: Impact of sector concentration on economic capital (EC_{sim}) for two sets of corporate credit portfolios and the total portfolio of the bank, based on factor correlations from Table 3

	EC only of corporate portfolio		EC of total bank portfolio	
	"High MEC"- rule	"Low- MEC"- rule	"High MEC"- rule	"Low- MEC"- rule
Benchmark portfolio	7.7	7.7	8.0	8.0
Portfolio 1	8.7	7.2	8.3	8.0
Portfolio 2	9.3	7.7	8.5	8.5
Portfolio 3	10.1	8.8	8.7	9.3
Portfolio 4	10.3	9.5	8.8	9.7
Portfolio 5	10.7	9.9	8.9	10.3
Portfolio 6	11.7	11.6	9.2	11

Table 19: Sector distribution of a weakly granular portfolio of 11 sectors, sorted by decreasing exposure size in each sector

Sector	Total Exposure	Exposure number · single exposure size			
1	11	1 · 11			
2	361	7 · 47	1 · 32		
3	692	5 · 120	1 · 92		
4	2020	16 · 120	1 · 100		
5	429	9 · 47	1 · 6		
6	898	7 · 120	1 · 58		
7	389	8 · 47	1 · 13		
8	545	1 · 120	1 · 110	6 · 47	1 · 33
9	192	4 · 47	1 · 4		
10	63	1 · 47	1 · 16		
11	400	8 · 47	1 · 24		

Table 20: Sector distribution of a weakly granular portfolio of eleven sectors, not sorted by exposure size in each sector

Sector	Total Exposure	Exposure number · single exposure size					
1	11	1 · 11					
2	361	3 · 47	1 · 32	4 · 47			
3	692	2 · 120	1 · 92	3 · 120			
4	2020	7 · 120	1 · 100	9 · 120			
5	429	4 · 47	1 · 6	5 · 47			
6	898	3 · 120	1 · 58	4 · 120			
7	389	4 · 47	1 · 13	4 · 47			
8	545	2 · 47	33	120	3 · 47	110	47
9	192	2 · 47	1 · 4	2 · 47			
10	63	1 · 47	1 · 16				
11	400	4 · 47	1 · 24	4 · 47			

Table 21: Sector distribution of a weakly granular single-sector portfolio

Sorted by exposure	Total Exposure	Exposure number · single exposure tupel			
Sorted	6000	32 · 120	45 · 47	1 · 45	
Unsorted	6000	1 · 45	1 · {120, 47}	15 · {47,120,47}	1 · 120

Table 22: Quality distribution of German firms in the Bundesbank database

Rating grade	AAA	AA	A	BBB	BB	B
Share in percent	2	6	11	55	24	2
PD in percent	0.01	0.02	0.07	0.26	0.87	3.27