Collateral and Debt Maturity Choice: A Signaling Model

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Keywords: Debt Maturity, Asymmetric Information, Signaling, Collateral

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ABSTRACT

This paper derives optimal loan policies under asymmetric information where banks offer loan contracts of long and short duration, backed or unbacked with collateral. The main novelty of the paper is that it analyzes a setting in which high quality firms have collateral as a complementary device along with debt maturity in signaling their superiority. The least-cost signaling equilibrium depends on the relative costs of the signaling devices, the difference in firm quality and the proportion of good firms in the market. Model simulations suggest a non-monotonic relationship between firm quality and debt maturity with high quality firms having both long-term secured debt and short-term secured or non-secured debt.

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1. INTRODUCTION

The role of asymmetric information on debt maturity choices has been the subject of a debate for quite some time, both in the theoretical and empirical literature. Among other theoretical studies in this field (Robbins and Schatzberg, 1986; Kale and Noe, 1990 and Diamond, 1993), the models by Flannery (1986) and Diamond (1991) emphasize the signaling properties of the debt maturity choice. In both models, firms have private information about their two-period projects and firms may signal their quality by borrowing on a short-term basis. However, there are also considerable differences. Flannery analysis a complete contracting model for high and low quality firms, while Diamond considers an incomplete contracting model for firms with different risk ratings. Moreover, unlike Flannery's model, the model by Diamond assumes short-term liquidity risk. The empirical implications of both models also differ. Flannery's model predicts debt maturity to be positively related to risk ratings: the high risk firms will borrow on a long-term basis, whereas the low risk firms will use short term debt. The model by Diamond, on the other hand, predicts a nonmonotonic relationship between firm quality and debt maturity. In his model, the extremely risky firms do not have access to long term debt and need to borrow short, the intermediate-risky firms will borrow long and the low risk firms will borrow short.

The empirical literature provides some support for the theoretical models of Flannery and Diamond. Consistent with the prediction of Flannery and Diamond, Berger et.al. (2004), Stohs and Mauer (1996), and Barclay and Smith (1995) find that firms with high bond ratings tend to use more short-term debt while firms with low bond ratings tend to have more long-term debt and firms without ratings have short-term debt. However, in contrast to the empirical predictions of both models, several empirical papers demonstrate that high quality firms do borrow on a long-term basis. For instance, Scherr and Hulbert (2001), using an accounting measure-Altman Z-score-to proxy for credit quality, find that high quality firms borrow on both a long- and short-term basis whereas low quality firms are restricted to long-term debt only. Furthermore, Molina and Penas (2004) provide evidence in favor of high quality firms using long-term debt. Thus, unlike the predictions of the main signaling debt maturity models, the empirical literature suggests that high quality firms may borrow on both a short- and long-term basis [1].

This papers contributes to the literature on signaling and debt maturity choice. In line with Flannery's and Diamond's work, our model considers a two-period asymmetric information setting between firms of different quality and a perfectly competitive bank. However, in contrast to all models we are aware of we allow firms to signal with two debt instruments. Specifically, we analyze the case where firms have the possibility to signal with collateral, in addition to debt maturity. In practice debt contracts often contain clauses regarding both debt maturity and collateral, our analysis therefore deals with an important policy issue, and is more in accordance with the observed regularities than the models analyzing a single signaling instrument[2]. Our aim is to derive optimal loan policies under asymmetric information where banks offer loan contracts of long and short duration, backed or unbacked with collateral[3]. Our model gives a theoretical backing for a wide range of empirical outcomes. In line with Diamond and Flannery, our model predicts the most risky firm to borrow on a long-term basis without collateral. However, for the other type of firms our model provides a justification for borrowing short-term debt, with or without collateral, and borrowing long-term debt without collateral. Thus, our analysis provides a broader justification for empirical regularities than most existing models.

We show that the choice for and relevance of the use of a particular signaling instrument or the use of two signaling instruments at the same time, depends e.g. on the proportion of good firms in the market, the difference in quality of the firm willing to signal and the most risky firm in the market, and the relative costs of the signaling possibilities available. To better explain the empirical implications of our model, we describe a possible outcome based on a model simulation. This simulation provides evidence suggesting a nonmonotonic relationship between firm quality and debt maturity. For a particular parameter setting, we show that the most risky firm will borrow long without collateral, firms that are slightly less risky and the group with the lowest risk firms will borrow long with collateral, and the intermediate-risky firms will borrow short with or without collateral. Most importantly, our analysis shows that the resulting equilibrium depends on a combination of a wide range of parameters, and therefore can not be described by a simple rule, such as high quality firms will borrow short and low quality firms will borrow long. The crux of the matter is that the choice for short or long term debt also depends on the availability and costs of other signaling instruments. This seems to be obvious, but has never been taken into account in the existing empirical and theoretical debt maturity literature.

The paper is organized in 6 sections. Section 2 provides a general outline of the model. Section 3 derives the bank's optimal loan strategy in a full information setting. Section 4 introduces asymmetric information and examines the optimal loan policy. In Section 5 some empirical implications of the analyzes are set out. Section 6 summarizes our results and provides some areas for further research.

2. GENERAL OUTLINE OF THE MODEL

The model has three periods. At time t=0 entrepreneurs are endowed with a risky investment project which lasts for two periods. If the investment project is undertaken, all cash flows will occur at the end of period 2. Entrepreneurs do not have initial wealth, so that outside finance is needed. All investment projects require a unit of investment, and thus a unit of external finance. The projects can be financed with short-term (*s*) or long-term (*l*) debt. The maturity time ($m\varepsilon(l,s)$)

in the model should be interpreted as being defined relative to the timing of the cash flows, rather than in terms of calendar time (see Diamond, 1991). Firms need to pay an additional amount of fixed transaction cost *b* if they issue short-term debt instead of long-term debt.

There are two types of borrowers, good (*g*) and bad (*b*), who differ in their "up" probabilities, so $i\ \varepsilon(g,b)$. The proportion of safe borrowers is equal to θ. Before the contract is signed the bank only knows the distribution of borrowers (i.e. the bank knows that a proportion θ of the borrowers are safe borrowers), but the probability of success of a particular borrower is private.

During each period there is a probability p_i that the project increases in value. At t=0 the bank and the entrepreneurs know that the projects' liquidating value at t=2 will be M_3 with probability p^2, M_4 with probability $2p(1-p)$ and 0 with probability $(1-p)^2$.

The time profile of the project's value is described in Figure 1.

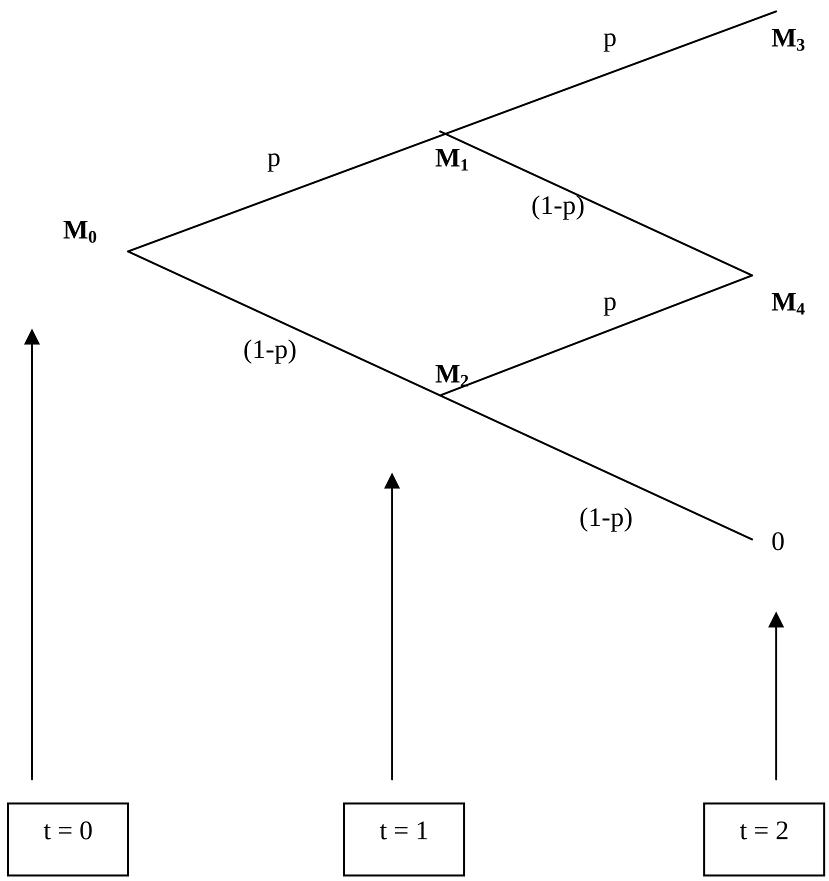

Figure 1: time profile of the model

In addition to the interest factor (one plus the loan rate: R_i), the perfectly competitive bank may demand collateral (C_i) in case the project's liquidation value is insufficient to repay the debt. Each borrower has a known end-of-period endowment *W*, which cannot be used to finance the investments, but can be used to pay the collateral. Firms face the cost of collateralization, which is proportional to the amount of collateral they post. This cost can be recognized as the costs of collection and marketing of the collateralized assets (Barro, 1976), or dissipative cost in liquidating collateral (Boot, Thakor, Udell, 1991) and are totally endured by firms as borrowers.

We assume the following:

A.1: $0<p_b<p_g<1$

A.2: Entrepreneurs and the bank are risk neutral

A.3: The bank makes zero profits

A.4: The riskless interest rate (the opportunity cost of capital per loan) is zero.

A.5: *M1*, *M2*, *M3*, and *M4* are larger than the promised debt repayments (principal plus interest).

A.6: The transaction costs (refinancing costs) of short-term debt equal *b*; the transaction costs of long-term debt equal 0.

A.7: Firms finance their investment projects with either short or long-term debt. We ignore the possibility of a mixture of the two forms of debt.

A.8: Firms face costs to put up collateral, as in Bester (1985). These costs are proportional to the amount of collateral by a factor *k*, so additional costs of collateralization equal *kC*.

A.9: $0 \le C_i < 1$

A.10: The end of period endowment *W* is high enough to pay off the needed collateral.

Most assumptions are straightforward. Some, however, need additional explanation. A.5 implies that we assume that all debt maturing at t=1 is riskless (no liquidity risk), which is in line with Flannery (1986). This assumption is made to not further complicate the model. An obvious drawback of this choice is that we ignore the possibility of short-term liquidation of the firm. Several papers (e.g. Diamond, 1991) emphasize that using short-term debt is advantageous since short-term debt may help to avoid strategic defaults by a threat of short-term liquidation of the firm. A.6 reflects the assumption that total transaction costs for short-term debt are higher than for long-term debt. The reason is that firms who decide to finance with short-term debt need to go more often (twice as much in our model) to a bank than firms who finance with long-term debt. The transaction costs *b* for short-term debt therefore can be interpreted as the additional costs of financing with short-term debt rather than with long-term debt. A.8 explains how we model the costly collateralization. Like Bester (1985), firms bear these costs and banks do not take them into account when setting an interest rate. In many papers, cost of collateralization is reflected by the disparity in value of collateral between banks and firms.[4] A.9 implies that we rule out the uninteresting case where loans can become entirely riskless if they are backed by collateral. Finally, A.10 implies that we assume that the available end-of-period wealth always exceeds the collateral requirement.

3. FULL INFORMATION

To provide a benchmark, we start by assuming that the bank can identify the quality of the borrowers without costs. From figure 1 we can derive the valuation of a borrower's equity if she borrows long or short.[5] This gives the following results.

If a firm i, i ε {g, b) borrows long the valuation of its equity (V_{li}) is equal to

$$(1)\quad V_{li} = p_i^2(M_3 - R_{lic} - kC_{li}) + 2p_i(1-p_i)(M_4 - R_{lic} - kC_{li}) - (1-p_i)^2(1+k)C_{li}$$

$$= p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(2-p_i)R_{li} - (1-p_i)^2 C_{li} - kC_{li}$$

where R_{li} is the loan rate on long run debt for borrower i.

If a firm i uses short-run debt, the valuation (V_{si}) is equal to

$$(2)\quad \begin{aligned} V_{si} &= p_i^2(M_3 - 1 - kC_{si}) + p_i(1-p_i)(M_4 - 1 - kC_{si}) \\ &+ p_i(1-p_i)(M_4 - R_{sic} - kC_{si}) - b - (1-p_i)^2(1+k)C_{si} \\ &= p_i^2 M_3 + 2p_i M_4(1-p_i) - p_i - (1-p_i)^2 C_{si} - p_i(1-p_i)R_{sic} - kC_{si} - b \end{aligned}$$

With perfect information, the bank knows the probability of success of the borrowers (the "up" probabilities). Under the zero profit constraints, the short and long term loan rates are given by:

(3) $1 = R_{li}\left[p_i^2 + 2p_i(1-p_i)\right] + (1-p_i)^2 C_{li} \Rightarrow R_{li} = \frac{1-(1-p_i)^2 C_{li}}{p_i(2-p_i)}$

(4) $1 = p_i R_{si} + (1-p_i)C_{si} \Rightarrow R_{si} = \frac{1-C_{si}(1-p_i)}{p_i}$

By substituting (3), and (4) in (1) and (2), respectively, the valuations for a firm *i* using short or long-term debt can be derived:

(5) $V_{li} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - 1 - kC_{li}$

(6) $V_{si} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - 1 - kC_{si} - b$

We also assume that the credit contracts are individually rational, i.e.,

A.11: $V_{im} > 0$ for $i \ \varepsilon \ \{g, b\}$ and $m \ \varepsilon \ \{l, s\}$

Proposition 1: Under A1-A11, the full information competitive equilibrium implies that both groups of firms borrow long without collateral. So, the full information equilibrium policy is given by $R_{li} = \frac{1}{p_i(2-p_i)}$; $C_i = 0$ and $m_i = l$.

Proof**:**

This solution is straightforward. The bank optimizes each type of borrower's expected utility subject to the zero profit constraints and the participation constraints. It is obvious that the bank's optimal policy will never imply that a firm borrows short or that the loan is backed by collateral since borrowing short and/ or securing a loan is costly. ■

4. ASYMMETRIC INFORMATION

We now assume that the bank does not know the type of the firm it faces. Before proceeding we explain the equilibrium concept we are using.

4.1 THE EQUILIBRIUM CONCEPT

In our model, we are concerned about *Perfect Bayesian Equilibria (PBE)*.[6] Since the *PBE* concept does not impose restrictions on out-of-equilibrium beliefs we follow the concept of the *Intuitive Criterion* (*IC*) of Cho and Kreps, (1987) to rule out perfect Bayesian equilibria that are upheld by unreasonable off-equilibrium beliefs.[7] In line with Spence (1973) and Riley (1979), the separating signaling equilibria we consider should satisfy the *Incentive Compatibility Constraint (ICC)* and the *Competitive Rationality Condition (CRC).* The *ICC* ensures that each agent is personally interested in accepting the contract designed for his type rather than the one designed for the other of agent. The *CRC* in our setting requires the credit market to be perfectly competitive and that banks have rational expectations so that in equilibrium they do not make profits and the loan interest rate correctly reflects firm's riskiness. In our model, there exists a continuum of separating equilibria, but the *IC* restricts the separating equilibria to *least-cost separating equilibria.* These equilibria are such that the bad firms do not signal and the good firms choose the minimum level of signaling that allows them to be separated without attracting the bad firms. These equilibria are the most efficient perfect Bayesian equilibria in that they entail the least wasteful signaling costs.

4.2 THE BORROWING POSSIBILITIES

Table 1. Different borrowing strategies.

Good\Bad	L with C	L without C	S with C	S without C
L with C	1: P	2: S	3: S	4: S
L without C	5: S	6: P	7: S	8: S
S with C	9: S	10: S	11: P	12: S
S without C	13: S	14: S	15: S	16: P

Notes: P (S) means a candidate pooling (separating) equilibrium. L denotes long-term debt; S denotes short-term debt; C denotes collateral.

Table 1 presents all possible borrowing choices for both types of firms. The optimal borrowing strategy of each type of firm depends on the behavior of the other type of firms, given his belief. If good firms succeed in signaling their quality by borrowing short, with or without collateral, or by borrowing long with collateral, a separating equilibrium may occur. However, bad firms may decide to mimic good firms and good firms can voluntarily decide not to signal their quality. By so doing, both groups of firms may opt for a pooling equilibrium if they achiever higher profits.

4.3 THE SET OF PERFECT BAYSIAN EQUILIBRIA

We identify the set of *PBE* by ruling out borrowing possibilities, which do not constitute a *PBE.* This will be the case when a firm prefers to deviate from the original borrowing strategy no matter what the bank believes. First, this rule allows us to discard immediately all separating possibilities where bad firms signal, depicted as cases 3, 4, 5, 7, 8, 9, 12, 13 and 15 in Table 1. If separation occurs, bad firms would always prefer not to signal to avoid signaling costs. Thus, for any contract that is a candidate for a separating equilibrium, the contract offered by the bad firm should coincide with his full information contract (see, for instance, Macho-Stadler and Perez-Castrillo, 2001, p. 203, Result 5.6). Second, we can also eliminate conceivable pooling equilibria for which bad firms have incentives to deviate to their full information contract. This may occur for all conceivable pooling equilibria with positive signaling costs. Thus, we derive conditions for which deviating to the full information contract provide bad firms with higher profits than pooling. If these conditions do not hold, separating equilibria do not exist. In order to make the necessary calculations, we need to derive expressions for the common interest rates if firms decide to pool. There are several possibilities. They may pool by both borrowing long or by both borrowing short, where in both cases they may back the loan by collateral. Under a pooling equilibrium, the bank believes that the proportion of good firms is θ and the proportion of bad firms is $1-\theta$. To comply with the *Competitive Rationality Condition*, the long *pooling* loan rate is given by:

$$
\begin{aligned}
(7)\qquad & 1=\theta R_{lp}\left[p_g^2+2p_g(1-p_g)\right]+\theta(1-p_g)^2C_{lp} \\
& +(1-\theta)R_{lp}\left[p_b^2+2p_b(1-p_b)\right]+\theta(1-p_b)^2C_{lp} \\
& \Rightarrow R_{lp}=\frac{1-C_{lp}\left[\theta(1-p_g)^2+(1-\theta)(1-p_b)^2\right]}{\theta(2-p_g)p_g+(1-\theta)(2-p_b)p_b}
\end{aligned}
$$

The *short pooling* loan interest rate is determined as follows. We know that short-term debt issued at t = 0 is riskless since all firms reach either M_1 or M_2. However, short-term debt issued at t=1 is subject to default risk if the borrowers decrease value at t=2. Since good and bad firms differ in their probabilities of reaching M_2, $(1-p_g)$ of the good firms and $(1-p_b)$ of the bad firms will arrive at state M_2. Therefore $\left[\frac{\theta(1-p_g)}{1-(\theta p_g+(1-\theta)p_b)}\right]$ of good firms and $\left[\frac{(1-\theta)(1-p_b)}{1-(\theta p_g+(1-\theta)p_b)}\right]$ of bad firms borrow at t = 1. Under *CRC* or the zero profit constraints, the loan rate then must satisfy

$$
(8)\qquad R_{sp}=\frac{1-(\theta p_g+(1-\theta)p_b)-C_{sp}\left[\theta(1-p_g)^2+(1-\theta)(1-p_b)^2\right]}{\theta(1-p_g)p_g+(1-\theta)(1-p_b)p_b}
$$

The subscript *p* denotes pooling.

Lemma 1: **(i**) the candidate pooling equilibrium where both groups of firms borrow long with collateral is not a perfect Bayesian equilibrium if

$$(9)\, C_{lp} \geq \frac{\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_b(2-p_b)}$$

(ii) The candidate pooling equilibrium where both groups of firms borrow short without collateral is not a perfect Bayesian equilibrium if

$$(10)\, b \geq (1-p_b)\left[1-p_b\frac{1-(\theta p_g+(1-\theta)p_b)}{\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)}\right].$$

(iii) The candidate pooling equilibrium where both groups of firms borrow short with collateral is not a perfect Bayesian equilibrium if

$$(11)\, C_{sp} \geq \frac{\theta(1-p_g)(1-p_b)(p_g-p_b)-b\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}{\theta(1-p_g)(1-p_b)(p_g-p_b)+k\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}$$

Proof: See the appendix.

The conditions (i), (ii) and (iii) determine when bad firms prefer separating long without collateral to pooling long with collateral, short without collateral and short with collateral, respectively. These conditions also provide the maximum levels of signaling costs for which bad firms can afford to pool with good firms under the three pooling possibilities. The three conditions are more likely to hold if the proportion of bad firms is very high (θ is low). In an extreme case, where θ

≈ 0, the conditions will always hold, irrespective of the other parameters, since the left-hand side of these expressions then become 0, 0 and –b/k, which are always smaller than the right-hand side of these expressions. For the remainder of the analysis, we introduce the above mentioned conditions as additional assumptions for the existence of separating signaling equilibria. We denote the conditions (i), (ii) and (iii) as A12, A13 and A14, respectively

A12. $C \geq C_{lp}^{*}$ where C_{lp}^{*} given by equation (9)

A13. $b \geq b_{p}^{*}$ where b_{p}^{*} given by equation (10)

A14. $C \geq C_{sp}^{*}$ where C_{sp}^{*} given by equation (11)

4.4 REFINEMENT OF *PBE* BY THE INTUITIVE CRITERION.

Given the above-defined restrictions, the set of *PBEs* is restricted to four borrowing strategies: pooling long without collateral, and three separating possibilities where bad firms do not signal and good firms can signal with different levels of collateral and short-term debt. By using the *IC* we further restrict the set of equilibria. We formulate the following lemma:

Lemma 2: The pooling equilibrium, where both firms borrow long without collateral does not satisfy the *Intuitive Criterion* if one of the following conditions hold:

i) $$\begin{cases} (1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{\lg} \end{cases}$$

ii) $$\begin{cases} p_b + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + b \end{cases}$$

iii) $$\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{sg} \end{cases}$$

where R_{lg}, R_{sg} and R_{lp} are given by equation (3), (4) and (7), respectively.

Proof: See the appendix.

4.5 INCENTIVE COMPATIBILITY CONSTRAINTS

Leaving out the borrowing possibilities that are not *PBE* and ignoring the pooling equilibrium that does not satisfy the *Intuitive Criterion*, we are endowed with the set of candidate separating *PBE* presented by cases 2, 10 and 14 in table 1. In equilibrium, good firms may exercise one of the following options to signal: borrowing long-term debt with collateral (separation 1), borrowing short-term debt without collateral (separation 2), or borrowing short-term debt with collateral (separation 3). A firm will choose the most efficient separating possibility that gives the highest profits, under the condition that A1-A14 hold and that the separating is incentive compatible.[8]

We denote $\hat{e}$(g) and $\hat{e}$(b) as the optimal contract chosen at equilibrium by good and bad firms, respectively and $\hat{V}$ (g) and $\hat{V}$ (b) as the profits at equilibrium for good firms and bad firms, respectively. The *ICCs* are formulated as:

$$U(\hat{e}(g), g, \hat{V}(g)) \geq \arg\max EU[e(b), g, V(b)]$$
$$U(\hat{e}(b), b, \hat{V}(b)) \geq \arg\max EU[e(g), b, V(g)]$$

The mimicking profit equations for firms of both types under the three separation possibilities are as follows:

$$(12) V_{lbc}^{mimic} = p_b^2 M_3 + 2 p_b M_4 (1 - p_b) - (1 - p_b)^2 C_{\lg} - p_b (2 - p_b) R_{\lg} - kC_{\lg}$$

$$(13) V_{sbcn}^{mimic} = p_b^2 M_3 + 2 p_b M_4 (1 - p_b) - p_b - p_b (1 - p_b) R_{sg} - b$$

$$(14) V_{sbc}^{mimic} = p_b^2 M_3 + 2 p_b M_4 (1 - p_b) - p_b - (1 - p_b)^2 C_{sg} - p_b (1 - p_b) R_{sg} - kC_{sg} - b$$

$$(15) V_{lgcn}^{mimic} = p_g^2 M_3 + 2 p_g M_4 (1 - p_g) - p_g (2 - p_g) R_{lb}$$

Where V_{lbc}^{mimic}, V_{sbcn}^{mimic} and V_{sbc}^{mimic} represent profits of bad firms when mimicking the behavior of good firms under the three separation possibilities and

$V_{\lg cn}^{mimic}$ indicates profits of good firms when pretending to be bad firms. The *ICCs* imply:

$$\begin{cases} V_{lbcn} > V_{lbc}^{mimic} \\ V_{\lg c} > V_{\lg cn}^{mimic} \end{cases} \text{ for separation (1),}$$

$$\begin{cases} V_{lbcn} > V_{sbcn}^{mimic} \\ V_{s\mathrm{g}cn} > V_{\lg cn}^{mimic} \end{cases} \text{ for separation (2)}$$

and $$\begin{cases} V_{lbcn} > V_{sbc}^{mimic} \\ V_{s\mathrm{g}c} > V_{\lg cn}^{mimic} \end{cases} \text{ for separation (3)}$$

With V_{lbcn} and V_{lgc} referring to profits of bad firms and good firms under separation. We rewrite the above expressions as:

$$(16) \begin{cases} (1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1 + kC_{\lg} \end{cases} \quad \text{for separation (1),}$$

$$(17) \begin{cases} p_b + p_b(1-p_b)R_{s\mathrm{g}} + b > 1 \\ p_g(2-p_g)R_{lb} \geq 1 + b \end{cases} \quad \text{for separation (2)}$$

$$(18) \begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{s\mathrm{g}} + b > 1 \\ p_g(2-p_g)R_{lb} \geq 1 + kC_{sg} + b \end{cases} \quad \text{for separation (3)}$$

To derive feasible incentive compatible separating equilibria, we should also consider the conditions for the existence of separation. More specifically, combining the conditions implied by the *IC*, which are given by Lemma 2, and the *ICCs*, we establish the following lemma.

Lemma 3: The feasible incentive compatible separating equilibria require the following conditions to hold

i) $\begin{cases} (1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} \geq 1 \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{\lg} \end{cases}$ for separation (1),

ii) $\begin{cases} p_b + p_b(1-p_b)R_{sg} + b > 1 \\ p_g(2-p_g)R_{lp} \geq 1 + b \end{cases}$ for separation (2)

iii) $\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b > 1 \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{sg} + b \end{cases}$ for separation (3)

<u>Proof</u>: See the appendix.

Lemma 3 implies the following for each separation.

For separation 1:

(19) $k_d \leq k \leq k_u$ with $$k_d = \frac{\left[(1-p_b)^2 - (1-p_g)^2\right](1-C_{\lg})}{p_g(2-p_g)C_{\lg}}$$ $$k_u = \frac{p_g(2-p_g)R_{lp} - 1}{C_{\lg}}$$

From condition (19), we can derive the minimum level of collateral for incentive compatibility to hold:

$$(20)\ C_{lgmin} = \frac{(1-p_b)^2 - (1-p_g)^2}{p_g(2-p_g)(1+k) - p_b(2-p_b)}$$

Note that this level of collateral satisfies assumption A12: $C_{lgmin} > C_{lp}*$. Proof: see the appendix.

If separation 1 occurs, good firms minimize signaling costs if they offer the minimum level of collateral, C_{lgmin}. If they do so, the lower bound of condition (19) will automatically be satisfied since then $k_d = k$. A feasible solution also requires that the upper bound of condition (19) exceeds the lower bound. Given that they choose C_{lgmin}, it can be derived that separation 1 may occur if:

$$(21)\ \theta \leq \theta_1 = \frac{p_g(2-p_g) - p_b(2-p_b)(1-C_{lg\min})}{p_g(2-p_g) + \left[p_g(2-p_g) - p_b(2-p_b)\right](1-C_{lg\min})}$$

For separation (2) the following needs to hold:

(22) $b_d < b < b_u$, with $$b_d = \frac{(p_g - p_b)(1-p_b)}{p_g}$$ $$b_u = p_g(2-p_g)R_{lp} - 1$$

Expression (22) implies that costs of short-term debt should be higher than a certain threshold to make it unattractive for bad firms to mimic good firms, and should be lower than another threshold to induce good firms to be truthful. Note that b_d is always greater than b_p^* given by assumption A13. <u>Proof</u>: see the appendix.

Condition (22) is feasible if $b_d < b_u$. This implies:

(23) $$\theta \leq \theta_2 = \frac{p_g^2(2-p_g) - p_b(2-p_b)\left[(p_g - p_b)(1-p_b) + p_g\right]}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left[(p_g - p_b)(1-p_b) + p_g\right]}$$

Thus separation 2 may result if (22) holds. A necessary (but not sufficient) condition for this is that $\theta < \theta_2$.

For separation (3) the following needs to hold:

(24) $$b_d^* \leq b \leq b_u^* \quad \text{with} \quad \begin{aligned} b_d^* &= b_d - C_{sg}\left[k + b_d\right] \\ b_u^* &= b_u - kC_{sg} \end{aligned}$$

Note that condition (24) allows lower values of b as compared with condition (22). This can be explained by the fact that good firms put up collateral and issue costly short-term debt at the same time under this separation. Therefore, even for $b < b_d$ separation may be incentive compatible since firms now also signal with collateral. From condition (24), we can derive the minimum level of collateral that good firms need to offer for this separation:

$$(25)\ C_{sgmin} = \frac{b_d - b}{b_d + k}$$

Equation (25) clearly shows that the minimum level of collateral needed to make separation incentive compatible decreases if the costs of issuing short-term debt increase. Note that $C_{sgmin} > C_{sp}*$, given by assumption A14, irrespective of all parameters. Proof: see the appendix.

If separation 3 occurs, good firms will offer C_{sgmin}. This guarantees that the lower bound of condition (24) will be fulfilled since then $b_d^* = b$. To make the condition feasible, the upper bound should exceed the lower bound i.e., $b_d^* \le b_u^*$. This requires:

$$(26)\ \theta \le \theta_3 = \frac{p_g(2-p_g) - p_b(2-p_b)\left[1 + b_d(1 - C_{sg\min})\right]}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left[1 + b_d(1 - C_{sg\min})\right]}$$

Note that for $b < b_d$, $\theta_3 > \theta_2$ independent of the other parameter values. The reverse holds when $b > b_d$. This specification is necessary to distinguish among different separation possibilities, as will be analyzed shortly.

4.6 THE LEAST-COST SEPARATING EQUILIBRIUM

The final step in the analysis is to solve for the least-cost separating equilibrium. It should be noticed that a separating equilibrium will only exist if the proportion of bad firms in the market is sufficiently high, $\theta < \max[\theta_1, \theta_2, \theta_3]$. Moreover, the conditions derived above imply that it is impossible that both separations (2) and (3) are feasible. Separation 3 will only be incentive compatible (for positive collateral values) if $b<b_d$ whereas separation 2 requires $b_d<b<b_u$. However, the conditions specified above may hold both for separation 1 and separation 2, or separation 1 and separation 3. The least-cost separating equilibrium then determines the outcome.

We determine the least-cost separating equilibrium by comparing the profits of good firms under the different separating equilibria. Profits of good firms under the three separating possibilities can be derived from equations (5) and (6). They are, respectively:

$$V_{\lg c} = p_g^2 M_3 + 2 p_g (1 - p_g) M_4 - 1 - kC_{\lg \min}$$
$$V_{sgcn} = p_g^2 M_3 + 2 p_g M_4 (1 - p_g) - 1 - b$$
$$V_{sgc} = p_g^2 M_3 + 2 p_g M_4 (1 - p_g) - 1 - kC_{sg\min} - b$$

where C_{lgmin} and C_{sgmin} are given by equation (20) and (25). We can now formulate the following Lemma.

Lemma 4:

(i) Given A1 – A14, and if parameter values are such that separation long with collateral and separation short without collateral are both incentive compatible, good firms will opt to separate by borrowing short if $b < kC_{lgmin}$

(ii) Given A1 – A14, and if parameter values are such that separation long with collateral and separation short with collateral are both incentive compatible, good firms will opt for separation short with collateral if $b + kC_{sgmin} < kC_{lgmin}$ or $b < k\left[\frac{C_{\lg\min}(b_d + k)}{b_d} - 1\right]$

From lemma 4 it follows that $\theta_1 < \theta_2$ and $\theta_1 < \theta_3$.

Proof : see the appendix.

Proposition 2: Under A1-A14, different separating equilibria may occur conditionally on the following parameter restrictions.

- Separation 1 occurs if

C1) $\theta < \theta_1$ and

C2) $b > \max[b_d, kC_{lgmin}]$ or

C3) $k\left[\frac{C_{lgmin}(b_d+k)}{b_d}-1\right]<b<b_d$

The least-cost separating signaling equilibrium is then characterized as follows:

C_{lb}=0; $C_{lg}=C_{lgmin}=\frac{(1-p_b)^2-(1-p_g)^2}{p_g(2-p_g)(1+k)-p_b(2-p_b)}$; m_g=l; m_b=l;

$R_{lg}=\frac{1-(1-p_g)^2 C_{lgmin}}{p_g(2-p_g)}$ and $R_{lb}=\frac{1}{p_b(2-p_b)}$

- Separation (2) occurs if

C4) $b_d < b < b_u$ and $\theta_1 < \theta < \theta_2$ or

C5) $\theta < \theta_1 < \theta_2$ and $b_d < b <$ min[kC_{lgmin}, b_u]

The least-cost separating signaling equilibrium is then described by

C_b=0; C_g=0; m_g=s; m_b=l; $R_{sg}=\frac{1}{p_g}$ and $R_{lb}=\frac{1}{p_b(2-p_b)}$.

- Separation (3) occurs if

C6) $\theta < \theta_3$ and b $<\min\left[k\left[\frac{C_{lgmin}(b_d+k)}{b_d}-1\right], b_d\right]$

If C6 hold, the least cost separating equilibrium implies:

C_b=0; C_g=$C_{sgmin} = \frac{b_d - b}{b_d + k}$; m_g=s; m_b=l; $R_{sg} = \frac{1-(1-p_g)C_{sgmin}}{p_g}$ and

$$R_{lb} = \frac{1}{p_b(2-p_b)}.$$

It is worthwhile to further discuss proposition 2. C1 implies that separation (1) is feasible. Separation (1) will be the least cost separating equilibrium if, in addition to C1, both separation (2) and (3) are either infeasible or incur higher signaling costs than separation 1. C2 indicates that separation 3 is not feasible because $b >$ max (b_d, kC_{lgmin}) while separation 2 is more costly. On the contrary, C3 indicates that separation 2 is not feasible and separation 3 is more costly than separation 1.

(C4) implies that separation 2 is incentive compatible, automatically precluding separation 3. (C4) also rules out separation 1 because the values of θ are in excess of θ_1, making separation 2 the least cost separating equilibrium. Moreover, if both separation (1) and (2) are feasible, separation (2) has the lowest cost if costs of borrowing short are lower than borrowing long with collateral. This is guaranteed by (C5). Similar conditions for separation (3) to be the least cost separating equilibrium are given by (C6).

5. WEALTH CONSTRAINTS AND EMPIRICAL IMPLICATIONS

To better explain the empirical implications of our model, we describe a possible outcome based on a model simulation. This allows us to specifically examine the impact of an increase in the difference between firm quality (p_g – p_b), and the transaction costs of short-term debt, for given values of k and θ. The graph below displays certain outcomes given a particular parameter setting with k = 0.7, θ = 0.2 and p_b =0.49. Notice that in this case separation always exists for all possible values of b and (p_g-p_b). Separation 2 will be chosen in the area given by the b_u, b_d and kC_{lgmin} schedules. Separation 3 occurs in the area enclosed by the b_d and $kC_{lgmin}(b_d+k)/(b_d)-k$ areas. Finally, separation 1 results in the remaining area. The graph shows that for a given b (below the intersection of the kC_{lgmin} and b_d schedules), separation 1 will occur for either small or large values of (p_g – p_b). However, for intermediate values for (p_g – p_b), either separation 2 or separation 3 will take place. This can be explained as follows. For small values of (p_g – p_b), b_u and b_d are too small to make separation 2 and 3 feasible. For large values of (p_g – p_b), separation 2 and 3 can be feasible but apparently more expensive than separation 1 since the kC_{lgmin} and $kC_{lgmin}(b_d+k)/(b_d)-k$ schedules fall below b_d. This implies that two sorts of firms including slightly less risky firms and lowest risk firms will opt for secured debt at long maturities. Firms of intermediate risk will choose short maturities with or without collateral. Finally, the most risky firms are settled with long-term non-secured debt. In addition, the graph also demonstrates how the signaling cost influences the separation at equilibrium. For a given value of (p_g – p_b) (to the left of the intersection b_d and kC_{lgmin} schedules) good firms will separate by borrowing short with collateral for low values of b. A rise in b such that b is in excess of b_d, leads good firms first to separate by borrowing short without collateral. However, if the transaction costs of short term debt become very high, seapration 1 will outdo separation 2 since separation 2 appears to be either unfeasible or more costly than separation 1.

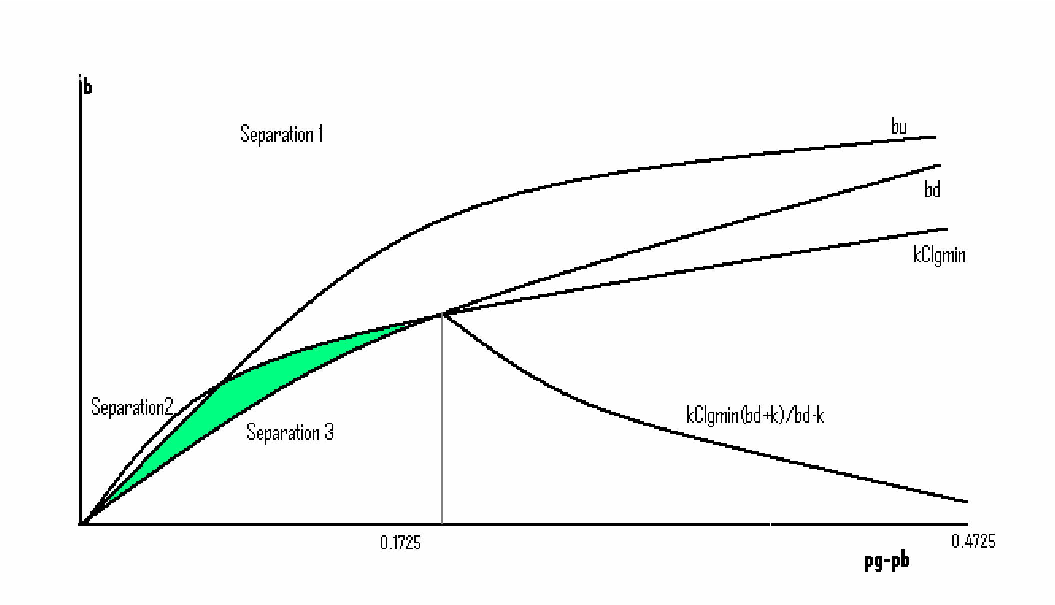

Figure 2. Different separating outcomes

In general, our analysis conveys several empirical implications. Firstly, we show that the resulting separating or pooling equilibrium depends on a combination of parameters.

1) θ: higher values make pooling more likely

2) *b:* higher values make signaling by long-term debt with collateral more likely

3) *k:* higher values make signaling by short-term debt without collateral more likely

4) (p_g – p_b): higher values increase the likelihood of separating since the threshold value for θ, below which one of the separating equilibria is feasible, rises.

Secondly, our signaling framework predicts a non-monotonic relationship between firm quality and debt maturity with high quality firms having both long-term secured debt and short-term secured or non-secured debt.

Overall, the analysis shows that the signaling outcome can not be described by a simple rule, such as high quality firms will offer collateral (as in the standard signaling models with collateral) or high quality firms will borrow short (as in the standard debt maturity signaling models). Rather, the use of a certain signaling device should be simultanously explored under the interactions with other signaling devices. Our model provides a theoretical justification for a broad range of possible optimal debt contracts for good firms to signal their qualuiry: they may borrow short, with or without collateral or they may borrow long with collateral. The decision depends on the relative costs of the signaling devices, the difference in firm quality and the proportion of good firms in the market.

Finally, we consider one simple extension of the model. In the analysis so far we have assumed that there is no wealth constraint. However, in the context of small firm financing in developing countries this does not seem to be realistic. A wealth constraint implies that the end-of –period endowment W is smaller than the needed collateral. From the previous analysis, we know that separation 1 requires good firms borrow long with collateral of C_{lgmin} and separation 3 requires good firms borrow short with collateral of C_{sgmin}, C_{lgmin} and C_{sgmin} are given by equation (20) and (25). It can be proved that $C_{sgmin} < C_{lgmin}$, irrespective of other parameters. Proof: see the appendix.

So, there will be a wealth constraint if $W < C_{lgmin}$. If $W < C_{sgmin} < C_{lgmin}$, good firms do not have a sufficient amount of collateral to be able to separate by borrowing long with collateral or short with collateral. Separation 2 turns out to be the only signaling option provided that the cost of short-term debt is sufficiently high. If $C_{sgmin} < W < C_{lgmin}$, good firms can still put up collateral to back their short-term debt, but not long-term debt. Therefore, separation 1 will never occur in equilibrium. Separation 2 or separation 3 will result conditionally on the cost of short-term debt and other parameter values.

6. CONCLUSION

This paper develops a model in which firms may signal with two debt attributes, duration and collateral. The analysis shows that different separating equilibria may result. If separation occurs, low-quality firms will always borrow long without collateral, while high-quality firms will borrow long with collateral or borrow short with or without collateral. The least-cost separating equilibrium depends on the relative signaling costs of the different signaling mechanism and the difference in firm quality. The analysis also indicates that separation will be more likely if the proportion of low-quality firms in the market is high. In addition, the simultaneous use of debt maturity and collateral as signaling devices plays a more significant role if the disparity in firm quality decreases. When a wealth constraint is imposed the role of collateral as a signaling device is undermined and high-quality firms have no other choice than to signal with short-term debt. This is probably the most relevant otcome for developing countries where wealth constraints are severe.

Model simulations suggest a non-monotonic relationship between firm quality and debt maturity with high quality firms having both long-term secured debt and short-term secured or non-secured debt. More importantly, the analysis clarifies that a proper empirical test of theoretical signaling models is not simple. Empirically testing the implications of signaling models at the least requires that the relative costs of the different signaling devices are taken into account. This has never been done, but certainly is an important area for future empirical research.

The model we have developed concentrates on the signaling properties of debt maturity and collateral. Further research aims to enrich the analysis by including other factors associated with the debt maturity and the collateral decision. It may, for instance, be interesting to introduce liquidity risk of short-term debt, allowing for a costly transfer of ownership in case of liquidation.

APPENDIX:

PROOF OF LEMMA 1

We will proof here that for the conditions specified in Lemma 1, bad firms prefer to deviate to the full information contract (the outcome for these firms if separation occurs), irrespective of bank beliefs. We consider the case where banks perceives a deviating action to be from firms of low quality. If bad firms prefer to deviate under this assumption, they will certainly do this for other bank beliefs (i.e. if the bank thinks that the deviating action is from good firms).

If both types of firms *pool by borrowing long with some amount of collateral*, the model has a continuum of pooling equilibria, which are all Pareto inferior. Suppose the equilibrium level of collateral is C_{lp}, where each C_{lp} in the interval [0, C_{lp}*], supports a different equilibrium. We can use this to find the value of C_{lp}^*, the greatest level of collateral that can be generated by a pooling equilibrium. The pooling equilibrium is defined as follows, where $C_{lp} \in [0, C_{lp}^*]$:

$$\begin{cases} m_g = m_b = l \\ C_g = C_b = C_{lp} \\ V_{lc}^{p} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(2-p_i)R_{lpc} - (1-p_i)^2 C_{lp} - kC_{lp} \\ \Pr ob(i = b \mid m = l, C = C_{lp}) = 1-\theta \\ \Pr ob(i = b \mid m = l, C \neq C_{lp}) = 1 \end{cases}$$

Bad firms prefer to deviate from the candidate pooling equilibrium if the following holds: $V_{1bcn} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - 1 > V_{lc}^p$

where V_{lbcn} denotes profits of bad firms under separation long without collateral, and V_{lb}^p denotes profits of bad firms at pooling long with collateral.

This implies that: $C_{lp} > C_{lp}^* = \dfrac{\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_b(2-p_b)}$

If both types of firms *pool by issuing short-term debt without collateral,* the candidate pooling equilibrium is defined as:

$$\begin{cases} m_g = m_b = s \\ C_g = C_b = 0 \\ V_{sbcn}^p = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i - p_i(1-p_i)R_{sp} - b \\ \Pr ob(i=b \mid m=s, C=0) = 1-\theta \\ \Pr ob(i=b \mid m=l, C=0) = 1 \end{cases}$$

Bad firms have incentives to deviate if pooling provides a lower value than the value of staying off-equilibrium: $V_{1bcn} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - 1 \geq V_{scn}^p$

where V_{lbcn} denotes profits of bad firms under separation long without collateral, and V_{scn}^p denotes profits of bad firms at pooling short without collateral.

The following is obtained: $b \geq b^* = (1-p_b)\left[1-p_b \frac{1-(\theta p_g + (1-\theta)p_b)}{\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)}\right]$.

Finally, if both types of firms *pool by borrowing short with collateral,* again the model allows for a continuum of pooling equilbria. Suppose the equilibrium level of collateral is C_{sp}, where each C_{sp} in the interval [0, C_{sp}*], supports a different equilibrium, the pooling is characterized:

$$\begin{cases} m_g = m_b = s \\ C_g = C_b = C_{sp} \\ V_{sic}^p = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(1-p_i)R_{spc} - p_i - (1-p_i)^2 C_{sp} - kC_{sp} - b \\ \Pr ob(i=b \mid m=s, C=C_{sp}) = 1-\theta \\ \Pr ob(i=b \mid m=l, C \neq C_{sp}) = 1 \end{cases}$$

We will determine C_{sp}* - the maximum level of collateral in equilibrium. Like the previous cases, bad firms prefer to deviate from the candidate pooling equilibrium if the following holds: $V_{1bcn} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - 1 > V_{sbc}^p$

where V_{lbcn} denotes profits of bad firms under separation long without collateral, and V_{sc}^p denotes profits of bad firms at pooling short without collateral.

or $C > C_{sp}^* = \dfrac{\theta(1-p_g)(1-p_b)(p_g-p_b) - b\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}{\theta(1-p_g)(1-p_b)(p_g-p_b) + k\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}$

For any value of C_{sp} greater than C_{sp}*, bad firms prefer deviating to pooling, irrespective of the bank's belief.

PROOF OF LEMMA 2

Consider the following characterization of the perfectly Bayesian pooling equilibrium:

$$\begin{cases} m_g = m_b = l \\ C_g = C_b = C_{lp} = 0 \\ \Pr ob(i = g \mid m = l, C = 0) = \theta \\ \Pr ob(i = g \mid m = s) = 0 \\ \Pr ob(i = g \mid C > 0) = 0 \\ V = V_{lcn}^{p} = p_i^2 M_3 + 2 p_i (1 - p_i) M_4 - p_i (2 - p_i) R_{lp} \quad \text{if } m = l \text{ and } C = 0 \\ V = V_b \quad \text{if } m = s \text{ and/or } C > 0 \end{cases}$$

we show that this pooling equilibrium does not satisfy the Intuitive Criterion.

In equilibrium, both types of firms pool by issuing non-secured long-term debt. So, no extra cost is incurred and signaling is uninformative. Rationally, the bank believes that the proportion of good firms is θ, hence they will charge the pooling loan rate of R_{lp} to all borrowing firms. If we assume that the bank perceives the willingness to borrow short and/or a placement of collateral to be from low quality firms and accordingly offers a loan contract designed for bad firms no firm will deviate from the pooling equilibrium, given the signaling cost they must pay and the lower profits they will obtain ($V_b < V_{lcn}^p$). Therefore, the pooling long without collateral appears as a *PBE* provided it is upheld by the bank's belief as specified

By introducing the *Intuitive Criterion*, we will consider whether or not such a belief is reasonable. If there exists parameters such that the specified bank's off-equilibrium-path belief is not intuitive, the perfect Bayesian pooling equilibrium does not survive the *Intuitive Criterion* and thus will be ignored.

Firms of any type have three options to deviate from pooling: (i) posting collateral to back their long-term debt, (ii) borrowing short-term debt without collateral; (iii) borrowing short-term debt with collateral. In doing so, they bear some deviating costs, which may be costs of short-term debt or cost of collateralization. As to bad firms, by deviating they wish to fool the bank to believe them to be of high quality. As to good firms, by deviating they wish to convince the bank to believe in their true quality. If bad firms are indifferent between pooling and deviating whereas good firms have incentives to deviate, it is reasonable for the bank to believe the deviating behavior to be from good firms. If so, the belief as specified appears unreasonable and the pooling is said to fail the *IC*. We will now show that this holds in our model under certain parameters.

First, we analyze the bad firms' behavior. Note that bad firms have the following options to deviate from pooling by mimicking Good firms and providing the bank with one of the signals: (i) posting collateral; (ii) borrowing short without collateral; (iii) borrowing short with collateral. They have to bear some deviating costs, which may be costs of short-term debt or cost of collateralization. In return, they will fool the bank to believe them to be of high quality. Bad firms' value with mimicking behavior are given by

$$(B1)\ V_{lbc}^{mimic} = p_b^2 M_3 + 2p_b M_4(1-p_b) - (1-p_b)^2 C_{\lg} - p_b(2-p_b)R_{\lg} - kC_{\lg}$$

$$(B2)\ V_{sbcn}^{mimic} = p_b^2 M_3 + 2p_b M_4(1-p_b) - p_b - p_b(1-p_b)R_{sg} - b$$

$$(B3)\ V_{sbc}^{mimic} = p_b^2 M_3 + 2p_b M_4(1-p_b) - p_b - (1-p_b)^2 C_{sg} - p_b(1-p_b)R_{sg} - kC_{sg} - b$$

The subscripts *lbc*, *sbcn*, *sbc* represent bad firms mimicking the behavior of good firms under the three above-mentioned alternatives. If bad firms pretend to be good firms, they will obtain the debt contract, i.e. the loan rate and the amount of collateral, designed for good firms. The interest rates R_{lg} and R_{sg} can be derived from equation (3) and (4).

If bad firms decide to pool, their value is:

$$(B4)\ V_{lbcn}^{p} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - p_b(2-p_b)R_{lp}$$

Bad firms are indifferent between pooling and deviating by mimicking good firms if the following holds.

$$(\text{B5})\begin{cases} V_{lbc}^{mimic} = V_{lbcn}^{p} \\ V_{sbcn}^{mimic} = V_{lbcn}^{p} \text{ or} \\ V_{sbc}^{mimic} = V_{lbcn}^{p} \end{cases}$$

$$\begin{cases} (B5a)(1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} > p_b(2-p_b)R_{lp} \\ (B5b)\, p_b + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ (B5c)\, p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \end{cases}$$

Second, what is the behavior of good firms? They may also deviate by signaling if they can benefit from it. Like bad firms, good firms also have three alternatives to deviate: i) posting collateral; (ii) borrowing short without collateral; (iii) borrowing short with collateral. Their values are equal to:

$$(\text{B6})\, V_{\lg c} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - kC_{\lg}$$

$$(\text{B7})\, V_{sgcn} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - b$$

$$(\text{B8})\, V_{sgc} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - kC_{sg} - b$$

If good firms pool, their value is:

$$(\text{B9})\, V_{\lg cn}^{p} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - p_g(2-p_g)R_{lp}$$

Good firms prefer to deviate if the following holds:

$$
\text{(B10)}\begin{cases} V_{\lg c} \geq V_{\lg cn}^{p} \\ V_{sgcn} \geq V_{\lg cn}^{p} \quad \text{or} \\ V_{sgc} \geq V_{\lg cn}^{p} \end{cases}
$$

$$
\begin{cases} (B10a)\, p_g(2-p_g)R_{lp} \geq 1+kC_{\lg} \\ (B10b)\, p_g(2-p_g)R_{lp} \geq 1+b \\ (B10c)\, p_g(2-p_g)R_{lp} \geq 1+kC_{s\mathrm{g}}+b \end{cases}
$$

This system of inequalities implies that the deviating options are more profitable than pooling.

If there exist parameters such that one equation in (B5) and its corresponding inequality in (B10) hold simultaneously, good firms are able to convince the bank that they are indeed better off by deviating than by staying on the equilibrium path. In order to support a deviating behavior by good firms, the reasonable off-equilibrium-path belief of the bank should be *Prob* $(i=g|m=s, C>0) = 1$. In other words, the off-equilibrium-path belief as specified in the definition of the pooling appears unreasonable. Hence, the pooling fails to meet the Cho-Kreps *Intuitive Criterion*.

Each equation in (B5) and its counterpart in (B10) hold simultaneously if the following conditions are satisfied:

$$
\text{i)} \begin{cases} (1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+kC_{\lg} \end{cases}
$$

ii) $\begin{cases} p_b + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+b \end{cases}$

iii) $\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+kC_{sg} \end{cases}$

Under any of the above conditions, a pooling long without collateral does not survive the *Intuitive Criterion*. Accordingly, it will be discarded from the set of *PBE*.

PROOF OF LEMMA 3

The feasible incentive compatible separating equilibria may result if both conditions implied by the Intuitive Criterion and the Incentive Compatibility Constraint are satisfied. We rewrite the IC given by Lemma 3, and the ICC given by equation (16), (17) and (18) as follows:

IC1) $\begin{cases} (1-p_b)^2 C_{lg} + p_b(2-p_b)R_{lg} + kC_{lg} > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+kC_{lg} \end{cases}$ for separation (1)

IC2) $\begin{cases} p_b + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+b \end{cases}$ for separation (2)

IC3) $\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+kC_{sg} \end{cases}$ for separation (3)

Under the *IC*, bad firms find pooling better than lying while good firms find separating more attractive than pooling.

$$ICC1)\begin{cases}(1-p_b)^2C_{\lg}+p_b(2-p_b)R_{\lg}+kC_{\lg}\geq 1\\ p_g(2-p_g)R_{lb}\geq 1+kC_{\lg}\end{cases}$$ for separation (1),

$$ICC2)\begin{cases}p_b+p_b(1-p_b)R_{sg}+b>1\\ p_g(2-p_g)R_{lb}\geq 1+b\end{cases}$$ for separation (2)

$$ICC3)\begin{cases}p_b+(1-p_b)^2C_{sg}+kC_{sg}+p_b(1-p_b)R_{sg}+b>1\\ p_g(2-p_g)R_{lb}\geq 1+kC_{sg}+b\end{cases}$$ for separation (3)

Under the *ICCs*, bad firms prefer truth-telling to lying whereas good firms prefer truth-telling to lying by pretending to be bad firms.

Now, the combination of both the *ICs* and the *ICCs* for each separation result in the following conditions:

$$\text{i)}\begin{cases}(1-p_b)^2C_{\lg}+p_b(2-p_b)R_{\lg}+kC_{\lg}\geq 1\\ p_g(2-p_g)R_{lp}\geq 1+kC_{\lg}\end{cases}$$ for separation (1),

$$\text{ii)}\begin{cases}p_b+p_b(1-p_b)R_{sg}+b>1\\ p_g(2-p_g)R_{lp}\geq 1+b\end{cases}$$ for separation (2)

$$\text{iii)}\begin{cases}p_b+(1-p_b)^2C_{sg}+kC_{sg}+p_b(1-p_b)R_{sg}+b>1\\ p_g(2-p_g)R_{lp}\geq 1+kC_{sg}+b\end{cases}$$ for separation (3)

Note that, the lower bound of each condition is derived from the *ICCs*, and the upper bound is derived from the *ICs*. For bad firms, pooling with good firms at borrowing long without collateral is better than presenting as themselves at separation. For good firms, pooling with bad firms is better than pretending to be bad firms at separation.

PROOF $C_{lgmin} > C_{lp}^{*}$

Since $C_{lp}^{*} = \dfrac{\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_b(2-p_b)}$ increases in θ and $C_{lp}^{*} = 0$ when $\theta = 0$ and $C_{lp}^{*} = C_{lgmin}$ when $\theta = 1$. So, $0 \leq C_{lp}^{*} \leq C_{\lg\min}$

PROOF $b_d > b_p^{*}$

We have $b_p^{*} = (1-p_b)\left[1-p_b\dfrac{1-(\theta p_g+(1-\theta)p_b)}{\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)}\right]$ in an increasing function of θ, $b_p^{*} = 0$ when $\theta = 0$ and $b_p^{*} = b_d$ when $\theta = 1$. This yields $b_p^{*} < b_d$.

PROOF $C_{sgmin} > C_{sp}^{*}$

Since $$C_{sp}^{*}=\frac{\theta(1-p_g)(1-p_b)(p_g-p_b)-b\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}{\theta(1-p_g)(1-p_b)(p_g-p_b)+k\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}$$

increases in θ and $C_{sp}^{*} < 0$ when $\theta = 0$ and $C_{sp}^{*} = C_{sgmin}$ when $\theta = 1$. So, $C_{sp}^{*} \le C_{sg\min}$

PROOF IF $b_d < kC_{lgmin}$ then $\theta_1 \le \theta_2$.

For simplicity, we rewrite the relevant expressions as follows:

$$C_{lgmin}=\frac{(1-p_b)^2-(1-p_g)^2}{p_g(2-p_g)(1+k)-p_b(2-p_b)} \qquad b_d=\frac{(p_g-p_b)(1-p_b)}{p_g}$$

$$\theta_1=\frac{p_g(2-p_g)-p_b(2-p_b)(1-C_{\lg\min})}{p_g(2-p_g)+\left[p_g(2-p_g)-p_b(2-p_b)\right](1-C_{\lg\min})}$$

$$\theta_2=\frac{p_g(2-p_g)-p_b(2-p_b)[1+b_d]}{\left[p_g(2-p_g)-p_b(2-p_b)\right][1+b_d]}$$

Inserting C_{lgmin} into θ_1, we obtain

$$\theta_1=\frac{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_g(2-p_g)}$$

Since θ_2 decreases in b_d, inserting $b_d \le kC_{lgmin}$, we have:

$$\theta_2 = \frac{p_g(2-p_g) - p_b(2-p_b)[1+b_d]}{\left[p_g(2-p_g) - p_b(2-p_b)\right][1+b_d]}$$

$$\geq \frac{p_g(2-p_g) - p_b(2-p_b)\left[1+kC_{lg\,\min}\right]}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left[1+kC_{lg\,\min}\right]}$$

$$= \frac{(1+k)\left[p_g(2-p_g) - p_b(2-p_b)\right]^2}{(1+k)\left[p_g(2-p_g) - p_b(2-p_b)\right]^2 + p_g(2-p_g)\left[p_g(2-p_g) - p_b(2-p_b)\right]k} = \theta_1$$

So $\theta_2 > \theta_1$ always holds if $b_d \leq kC_{lgmin}$

PROOF IF $b \leq k\left[\frac{C_{\lg\min}(b_d+k)}{b_d} - 1\right]$ then $\theta_1 \leq \theta_3$.

Recall expressions (25) and (26) with

$C_{sgmin} = \frac{b_d - b}{b_d + k}$ and $\theta_3 = \frac{p_g(2-p_g) - p_b(2-p_b)\left[1+b_d(1-C_{sg\,\min})\right]}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left[1+b_d(1-C_{sg\,\min})\right]}$

Inserting C_{sgmin}, we obtain

$\theta_3 = \frac{p_g(2-p_g) - p_b(2-p_b)[1+b_d(b+k)/(b_d+k)]}{\left[p_g(2-p_g) - p_b(2-p_b)\right][1+b_d(b+k)/(b_d+k)]}$ is the decreasing function in *b*.

Therefore, with $b = k\left[\frac{C_{\lg\min}(b_d+k)}{b_d} - 1\right]$,

$$\theta_3 = \theta_{3\min} = \frac{\left[p_g(2-p_g)-p_b(2-p_b)\right]^2(1+k)}{\left[p_g(2-p_g)-p_b(2-p_b)\right]\left\{\left[p_g(2-p_g)-p_b(2-p_b)\right](1+k)+kp_g(2-p_g)\right\}}$$

$$= \frac{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_g(2-p_g)} = \theta_1 .$$

So, $\theta_1 \le \theta_3$ for all $b \le k\left[\frac{C_{\lg\min}(b_d+k)}{b_d} - 1\right]$.

PROOF $C_{sgmin} < C_{lgmin}$

From the analysis we have

$C_{lgmin} = \frac{(1-p_b)^2-(1-p_g)^2}{p_g(2-p_g)(1+k)-p_b(2-p_b)}$ and $C_{sgmin} = \frac{b_d-b}{b_d+k}$ with

$$b_d = \frac{(p_g-p_b)(1-p_b)}{p_g}$$

We need to prove $C_{lgmin} > C_{sgmin}$ or $\frac{(1-p_b)^2-(1-p_g)^2}{p_g(2-p_g)(1+k)-p_b(2-p_b)} > \frac{b_d-b}{b_d+k}$

Rewrite this expression, the inequality becomes:

$$b\left[p_g(2-p_g)-p_b(2-p_b)+kp_g(2-p_g)\right] > k\left[\frac{(p_g-p_b)(1-p_b)}{p_g}p_g(2-p_g)-p_g(2-p_g)+p_b(2-p_b)\right]$$

$$= k(p_g-p_b)p_b(p_g-1)$$

Since the left-hand side is positive and right-hand side is negative, the inequality always holds.

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[1]For other empirical papers testing the choice of debt maturity as a signaling tool, see e.g. Guedes and Opler (1996), Mitchell (1993), and Scherr and Hulburt (2001).

[2] Since we focus on signaling properties of duration and collateral, we abstract from other factors associated with the debt maturity and the collateral decision. Therefore, we do not deal with e.g. tax-timing arguments of the debt maturity structure (see e.g. Brick and Palmon, 1992), maturity-matching arguments of duration, liquidity risk arguments of short-term debt (see Diamond, 1991) and the consequences of a firm's debt maturity decision on agency costs (see e.g. Myers, 1977). We also abstract from traditional trade-off theory arguments related to collateral, according to which a firms ability to obtain funds from banks is limited to the value of its collateralizable assets.

[3] The screening and signaling role of collateral has been well theoretically explored by Bester (1985, 1987), Besanko and Thakor (1987). See Coco (2000) for a more extensive survey.

[4] Some papers, e.g. Boot, Thakor and Udell (1991), assume that the dissipative costs of collateral are smaller for long-term debt than for short-term debt. The reason is that the bank has more timing flexibility in terms of when to force default with long-term debt than with short-term debt. However, this is based on the idea of renegotiating possibilities, which we ignore.

[5] Recall that long-term debt is two-period debt issued at t=0 and that short-term debt is one-period debt issued at t=0 and t=1. Moreover, recall that we assume that short-term debt issued at t=0 is riskless (so that the lending rate equals 1) and that short-term debt issued at t=1 is risky.

[6] A *PBE* is defined as a set of strategies and beliefs such that (Rasmusen, 1989, p. 146): 1) the strategies for the remainder of the game are Nash given the beliefs and strategies of the other players; 2) the beliefs at each information set are rational given the evidence appearing thus far in the game. This means that along the equilibrium path beliefs are based on priors updated by Bayes' Rule, if possible. Off the equilibrium path, Bayes updating is not possible since the deviating action is taken with probability zero in equilibrium.

[7] The *IC* restricts the beliefs out of equilibrium by requiring that the uninformed player's belief must put zero probability on an informed player who could not benefit from the off-equilibrium action no matter what beliefs were held by the bank. We use the *IC* to rule out unreasonable perfect Bayesian pooling equilibria. Note that in the setting of Rotschild and Stiglitz (1976) there cannot be a pooling Nash equilibrium under asymmetric information.

[8] Flannery (1986) does not examine whether the separating equilibrium is incentive compatible. He derives parameter restrictions for different types of equilibria (pooling and separating) by simply comparing profits for high-quality firms under different pooling possibilities with the profits for high-quality firms under a separating equilibrium. This may reveal that the actual conditions under which it becomes profitable for high-quality firms to separate themselves from low-quality firms by issuing short-term debt are much more restrictive than those given in Flannery (1986).